Course Title	Content Area	Grade Level	Credit (if applicable)
Grade 7 Math: Bridge to Algebra	**Mathematics**	**Grade 7**	**N/A**

Course Description

This course bridges student knowledge from the cumulative K-6 math experience to Algebra 1 in Grade 8. The course is a cohesive 9-unit sequence covering the final three units of Accelerated Grade 6 and the first six units of Accelerated Grade 7. This combined pathway is designed for students transitioning from regular Grade 6 to Accelerated Grade 7, ensuring they build essential foundations before accelerating.

This combined sequence emphasizes the development of proportional reasoning, rational number operations, transformational geometry, and algebraic thinking. It builds progressively from concrete representations to abstract reasoning. Students engage with problem-based learning that activates prior knowledge and builds toward increasingly sophisticated mathematical thinking. The course prioritizes multiple representations (visual, tabular, algebraic, contextual) to deepen conceptual understanding and prepare students for advanced mathematics.

Aligned Core Resources

CT Core Standards
(aligned to National Common Core Standards)

Imagine Learning iM Resources
(Imagine 6-8)
BPS teacher login through ClassLink required

Grade 6 ACC
https://accessim.org
https://accessim.org/6-8-accelerated/accelerated-6/course-guide/further-reading?a=teacher
Grade 7 ACC
https://accessim.org/6-8-accelerated/accelerated-7/course-guide/further-reading?a=teacher

- Empowering All Storytellers: Tips for Engaging Special Populations Using IM® v.360 for Grade 6–12
- Tackling Wordy Problems: How the Three Reads Math Language Routine Supports Access for All Learners
- Think Pair Share
- Making Sense of Story Problems
- Math Language Routines: Discourse with a Purpose
- Unlocking Learners' Thinking Using the Mathematical Language Routines

Connection to the *BPS Vision of the Graduate*

Common Core State Standards: Math Practice (MP) Standards

- MP 1: Make sense of problems and persevere in solving them.
- MP 2: Reason abstractly and quantitatively.
- MP 3: Construct viable arguments and critique the reasoning of others.
- MP 4: Model with mathematics.
- MP 5: Use appropriate tools strategically.
- MP 6: Attend to precision.
- MP 7: Look for and make use of structure.
- MP 8: Look for and express regularity in repeated reasoning.

	Lessons that Showcase Math Practice Standards								
	Unit 1 *6ACC U5*	**Unit 2** *6ACC U6*	**Unit 3** *6ACC U7*	**Unit 4** *7ACC U3*	**Unit 5** *7ACC U4*	**Unit 6** *7ACC U1*	**Unit 7** *7ACC U2*	**Unit 8** *7ACC U5*	**Unit 9** *7ACC U6*
MP 1	4, 6, 13, 18–20	1, 4, 5, 8, 10–12	1, 5, 8, 11, 17	1, 4, 5, 8, 10, 11, 17	-	1, 5, 12, 15	1, 9, 11, 18, 19	2, 4, 17, 18, 24	1, 8, 15, 19–21
MP 2	1–3, 7, 8, 13, 16, 19	2, 3, 6, 9, 13	1, 3, 4, 6, 8, 10, 12, 14, 17, 19–21	4, 5, 10, 12–14	5, 12	-	4, 19	1, 3–8, 10, 14, 17–20, 22–24, 26–28	3–7, 13, 14, 17, 18
MP 3	4, 11, 15–17, 19	1, 3, 5, 9–11, 13	2, 3, 9, 18, 20, 21	2, 3, 6, 9, 11, 13, 16, 18	2–4, 6–8	2, 10, 12, 16	1, 12, 13, 19	5, 8, 9, 12, 14–17	3, 11, 13
MP 4	7, 10, 14, 17, 18, 20	11	23	2, 17	12	-	18, 19	4, 5, 7, 8, 11, 14	10, 14
MP 5	11	-	-	-	-	6, 10	2, 4, 9, 10	28	-
MP 6	1, 2, 5–7, 9, 11, 13, 15, 16, 19	3–8, 10, 12	3, 5, 6, 11, 13, 15, 16, 19, 22	3, 7, 8, 13, 15, 17	1, 3, 4, 7, 12	1–3, 5, 9, 12, 17	1, 3, 7, 8, 10–12	1, 2, 8–10, 17, 19, 21, 22, 24	2, 5, 21
MP 7	3, 5, 8, 9, 12, 14–16	1–4, 6–8, 11	1, 2, 7, 9, 11, 15, 16, 18, 20	2, 4–6, 9, 12, 14, 16, 18	1–4, 6, 9–11	1, 3, 4, 7–9, 13–16	2, 5, 6, 8–10, 13, 15–17	2, 5, 7, 12, 15, 16, 18, 20, 23, 24	2, 10–13, 16, 19, 20
MP 8	1–3, 8, 10, 12, 15	2, 6–8	6, 7, 9, 10, 13–15	1, 15	10	7, 12, 13, 15	2, 3, 6, 14–17	6, 8, 9, 11	3, 23

Bristol Public Schools Vision of the Graduate

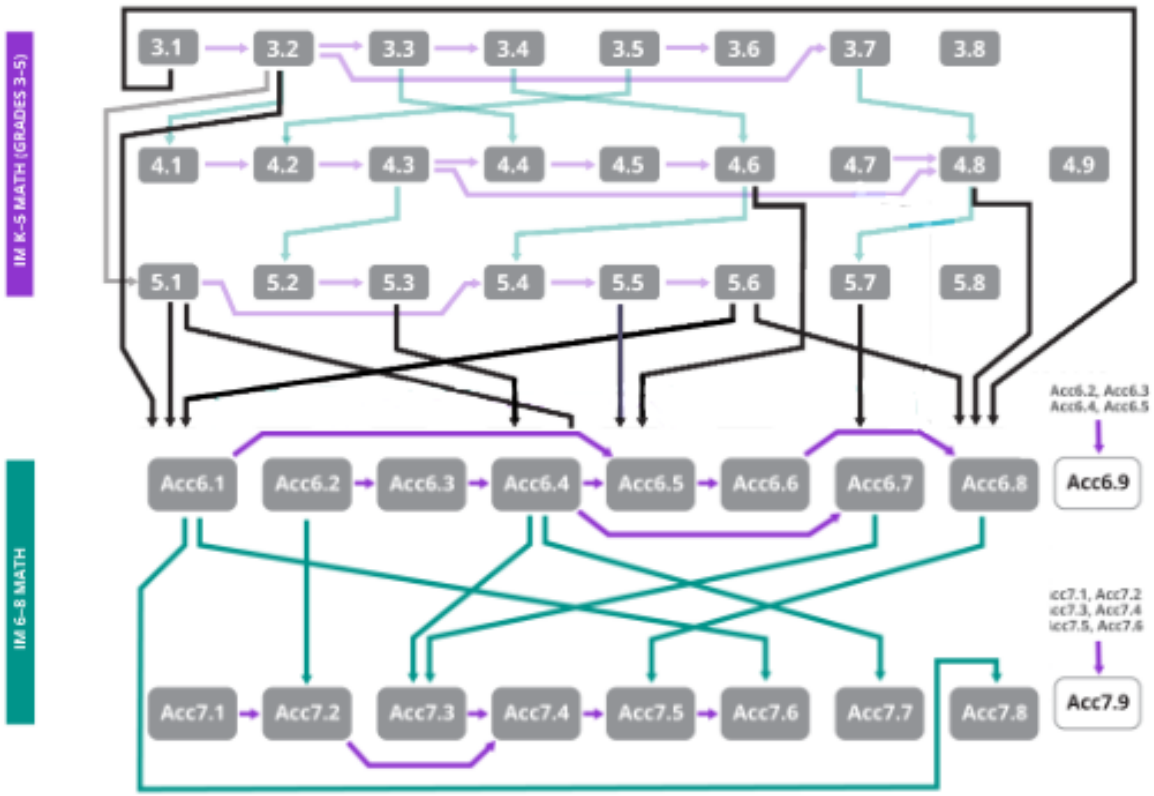
Problem Solving
- iM's focus on real-world modeling and problem-solving strategies
- Multiple solution pathways are encouraged and explored
- Students develop perseverance through challenging tasks

Critical Thinking
- Students analyze mathematical relationships and justify their reasoning
- Regular opportunities to critique others' reasoning
- Emphasis on understanding "why" not just "how"

Communication and Collaboration
- Structured mathematical discourse is built into lessons
- Students explain their thinking both verbally and in writing
- Many activities involve partner and group work

Link to *Equity Audit*	Equity Curriculum Review Audit (Gr. 7 Bridge)

Additional Course Information:
Knowledge/Skill Dependent courses/prerequisites



Standard Matrix

Standard	Unit 1 *6ACC U5*	Unit 2 *6ACC U6*	Unit 3 *6ACC U7*	Unit 4 *7ACC U3*	Unit 5 *7ACC U4*	Unit 6 *7ACC U1*	Unit 7 *7ACC U2*	Unit 8 *7ACC U5*	Unit 9 *7ACC U6*
6.G.A.3			22						
6.NS.C.5			1, 3						
6.NS.C.6			1, 3, 5, 14						
6.NS.C.6.a			3						
6.NS.C.6.b			11, 13						
6.NS.C.6.c			11, 12, 13						
6.NS.C.7			3, 4, 5						
6.NS.C.7.a			2, 5						
6.NS.C.7.b			2						
6.NS.C.7.c			4, 12						
6.NS.C.7.d			4, 5						
6.NS.C.8			12, 13, 22						
7.EE.A					5				
7.EE.A.1					1-4				
7.EE.A.2		2, 6, 8		11					
7.EE.B.3	20		17, 20 ,23	2-5, 10, 11					
7.EE.B.4			20, 21	5, 8, 10, 11, 15,18					

7.EE.B.4.a				4-11					
7.EE.B.4.b				14, 16, 17					
7.G.A	11								
7.G.A.1	14, 20						1-7, 18		
7.G.A.2	11					15-17, 22			
7.G.A.3									8, 10
7.G.B	14, 15								14
7.G.B.4	12, 13, 14, 16, 29, 20								
7.G.B.5				18		12			
7.G.B.6	14								9, 10, 13, 14, 22
7.NS.A			18, 20						
7.NS.A.1			1, 8		1				
7.NS.A.1.a			6-8						
7.NS.A.1.b			6, 7						
7.NS.A.1.c			9 ,10 ,13		1				
7.NS.A.1.d			7, 10 ,18						
7.NS.A.2.a			14, 17						
7.NS.A.2.b			16						
7.NS.A.2.c			14, 15, 18						
7.NS.A.2.d		3							
7.NS.A.3			10, 17, 19, 20, 21, 23						
7.RP.A	2, 6, 17		14, 17						14
7.RP.A.1	5	1							
7.RP.A.2	1-9. 12, 17, 18	1-3							
7.RP.A.2.a	4, 5, 10, 12, 15								
7.RP.A.2.b	2, 7, 9								
7.RP.A.2.c	1, 2, 4, 5, 9								
7.RP.A.2.d	7 9								
7.RP.A.3		4-13	23						
8.EE.B								1-11	
8.EE.B.5								2, 3, 5	
8.EE.B.6							15 - 17, 6, 9, 10, 26		
8.EE.C					5-8, 11, 12				
8.EE.C.7					6-9				
8.EE.C.7.a					10, 11				
8.EE.C.7.b					9				
8.EE.C.8								13-17	
8.EE.C.8.a								12, 14, 15, 26	
8.EE.C.8.b								14, 27	
8.EE.C.8.c								17, 27	

8.F.A									3, 23
8.F.A.1									1-4, 17
8.F.A.2									6, 7
8.F.A.3									4, 6, 7, 17, 18
8.F.B									17, 18
8.F.B.4									5, 7, 8
8.F.B.5									4, 6, 8
8.G.A						18	9-11, 13, 14, 17		14
8.G.A.1						2, 3,5, 10 ,12			
8.G.A.1.a						6-9, 11			
8.G.A.1.b						6-9			
8.G.A.1.c						8			
8.G.A.2						10, 11, 13			
8.G.A.3						4, 5	10, 11, 17		
8.G.A.4							12, 14		
8.G.A.5						12-14	13, 19		
8.G.C									17, 19, 20
8.G.C.9									11, 12, 15-21, 23
8.SP.A								28	
8.SP.A.1								18-23	
8.SP.A.2								20-23	
8.SP.A.3								19, 22, 23	
8.SP.A.4								24, 25	

Unit Links

[Grade 7 Mathematics: Bridge to Algebra]
[Unit 1: Proportional Relationships (G6 ACC U5)]
[Unit 2: Percent Increase and Decrease (G6 ACC U6)]
[Unit 3: Rational Numbers (G6 ACC U7)]
[Unit 4: Equations and Inequalities (G7 ACC U3)]
[Unit 5: Expressions and More Equations (G7 ACC U4)]
[Unit 6: Rigid Transformations & Congruence (G7 ACC U1)]
[Unit 7: Scale Drawings, Similarity, & Slope (G7 ACC U2)]
[Unit 8: Linear Relationships (G7 ACC U5)]
[Unit 9: Functions and Volume (G7 ACC U6)]
[Course Assessment Map]

Use of Instructional Time (181 School Days)

- ➔ 162 iM Content and Assessment Days
- ➔ 6 Climate and Culture Days: 2 days at start of year, 2 shortened days before breaks, and 2 days at end of year
- ➔ 9 IAB Days: 1 day Strategic Review and 2 day IAB in fall, winter, and spring
- ➔ 4 SBA Days: 1 day Strategic Review and 3 day SBA

Unit Title:

Unit 1: Proportional Relationships (G6 ACC U5)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	7.RP.A.2, 7.RP.A.2.c	Lesson 11	7.G.A, 7.G.A.2
Lesson 2	7.RP.A7.RP.A.2, 7.RP.A.2.b, 7.RP.A.2.c	Lesson 12	7.G.B.4, 7.RP.A.2, 7.RP.A.2.a
Lesson 3	7.RP.A.2	Lesson 13	7.G.B.4
Lesson 4	7.RP.A.2, 7.RP.A.2.a, 7.RP.A.2.c	Lesson 14	7.G.A.1, 7.G.B, 7.G.B.6
Lesson 5	7.RP.A.1, 7.RP.A.2, 7.RP.A.2.a , 7.RP.A.2.c	Lesson 15	7.G.B7.G.B.4, 7.RP.A.2.a
Lesson 6	7.RP.A , 7.RP.A.2	Lesson 16	7.G.B.4
Lesson 7	7.RP.A.2, 7.RP.A.2.b 7.RP.A.2.d	Lesson 17	7.RP.A, 7.RP.A.2
Lesson 8	7.RP.A.2	Lesson 18	7.RP.A.2
Lesson 9	7.RP.A.2, 7.RP.A.2.b, 7.RP.A.2.c, 7.RP.A.2.d	Lesson 19	7.G.B.4
Lesson 10	7.RP.A.2.a	Lesson 20	7.EE.B.3, 7.G.A.1, 7.G.B.4

Essential Question(s):

- How can a proportional relationship be identified and interpreted across tables, equations, and graphs?
- Why does every proportional relationship have two reciprocal constants of proportionality?
- How does the relationship between a circle's diameter and circumference illustrate a proportional relationship?

Enduring Understanding(s):

- A proportional relationship is defined by a constant ratio between quantities, appearing as a straight line through the origin on a graph and represented by an equation of the form y=kx
- Because a proportional relationship relates two quantities, there are two unit rates (e.g., cups per tablespoon and tablespoons per cup) that are reciprocals of each other
- The circumference of any circle is proportional to its diameter, and the constant of proportionality is always the value of pi (π)

Demonstration of Learning:

Checkpoint A is an opportunity for feedback
Checkpoint B is an opportunity for feedback
CFA 1: Checkpoint C (after lesson 9)
CFA 2: Checkpoint D (after Lesson 13)
CFA 3: Checkpoint E (after lesson 16)
EoU: Assessment A (after lesson 16)

Pacing for Unit

20 Days
Lesson Notes:

- Covering Lessons 1-16
- 2 days for review/assessment
- 2 flex days
- Optional Lessons: 17, 18, 19 & 20

Family Overview

https://accessim.org/6-8-accelerated/accelerated-6/unit-5?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
Acc6.5.1	constant of proportionality proportional relationship	___ is proportional to ___
Acc6.5.2	steady situation	reciprocal
Acc6.5.4		constant of proportionality proportional relationship

Aligned Unit Materials, Resources, and Technology

For whole course:
https://accessim.org/6-8/grade-7/course-guide/required-materials?a=teacher

Digital Applets

- Acc6.5.7 Notice These Points, T-Shirts for Sale
- Acc6.5.8 Race to the Bumper Cars, Space Rocks, and the Price of Rope
- Acc6.5.9 Tables, Graphs, and Equations, Hot Dog Eating Contest
- 6.5.10 Perimeter of a Square
- 6.5.11 Drawing Circles
- 6.5.12 Measuring Circumference and Diameter
- 6.5.13 Using pi
- 6.5.15 Estimating Areas of Circles, Making Another

Acc6.5.5		constant
Acc6.5.7	origin coordinate plane quantity axes	coordinates
Acc6.5.9	x-coordinate y-coordinate	origin
Acc6.5.10	perimeter	
Acc6.5.11	radius diameter circumference center (of a circle)	circle
Acc6.5.12	pi (π)	
Acc6.5.13	half-circle rotation approximation	diameter circumference pi (π)
Acc6.5.14	floor plan	approximate estimate
Acc6.5.15	area of a circle formula	radius
Acc6.5.16	in terms of π	area of a circle
Acc6.5.17		axes
Acc6.5.18	reasonable	
Acc6.5.19		squared center (of a circle) formula
Acc6.5.20	design	

Polygon out of a Circle

Provide access as needed throughout the unit:

- Blank paper
- Colored pencils
- Compasses
- Cylindrical household items
- Empty toilet paper roll
- Four-function calculators
- Geometry toolkits (Tracing paper, graph paper, colored pencils, scissors, an index card to use as a straightedge or to mark right angles, a ruler, and a protractor. Clear protractors with no holes and with radial lines printed on them are recommended)
  Notes: (1) "Tracing paper" is easiest to use when it's smaller. Commercially available "patty paper" is 5 inches by 5 inches and is ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required, they are listed as separate Required Material.
- Glue or glue sticks
- Internet-enabled device
- Markers
- Math Community Chart
- Measuring tapes
- Rulers
- Rulers marked with centimeters
- Scissors
- Snap cubes
- Tools for creating a visual display
  Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Lesson	Materials to Gather	Materials to Copy
4	• Four-function calculators: Lesson • Math Community Chart: Activity 2	
5	Snap cubes: Activity 2	
6		Biking and Rain Cards (1 copy for every 4 students): Activity 1
7	Rulers: Activity 1	
8	• Colored pencils: Activity 1 • Rulers: Activity 1	
9	Rulers: Activity 1	Tables, Graphs, and Equations Handout (1 copy for every 3 students): Activity 1
10	• Rulers marked with centimeters: Activity 1 • Four-function calculators: Activity 2	Perimeter of a Square Handout (1 copy for every student): Activity 1
11	• Compasses: Activity 3 • Rulers: Activity 3	Sorting Round Objects Cards (1 copy for every 2 students): Activity 1
12	• Empty toilet paper roll: Warm-up • Cylindrical household items: Activity 1	

		• Measuring tapes: Activity 1	
	13	Four-function calculators: Activity 1	
	14	Geometry toolkits: Lesson, Activity 1	
	15	• Geometry toolkits: Activity 1 • Blank paper: Activity 2 • Glue or glue sticks: Activity 2 • Markers: Activity 2 • Scissors: Activity 2	• Estimating Areas of Circles Handout (1 copy for every 12 students): Activity 1 • Making a Polygon out of a Circle Cutouts (1 copy for every 12 students): Activity 2
	17	Tools for creating a visual display: Activity 1	Creating and Representing Situations Handout (1 copy for every student): Activity 1
	18	• Internet-enabled device: Activity 1 • Tools for creating a visual display: Activity 2	
	19	Math Community Chart: Activity 1	• Circle Problems Cards (1 copy for every 2 students): Activity 1 • Visual Display of Circle Problem Handout (1 copy for every 10 students): Activity 2 • Merry-go-round and Unicycle Cards (1 copy for every 4 students): Activity 4
	20	• Blank paper: Activity 3 • Compasses: Activity 3 • Geometry toolkits: Activity 3	

Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions:**
• **Geography**: Students use scale drawings and maps to calculate actual real-world distances • **Art and Design:** Students explore scaling in the context of printing portraits and analyzing the proportions of various artistic figures • **Biology:** Scaling is explored through the lens of "Movie Monsters," examining how size changes affect physical properties	**Unit Rate Reversal:** Students may find the reciprocal unit rate (e.g., cups per tablespoon instead of tablespoons per cup) and apply it incorrectly. See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:	**Connections to Future Units:**
Essential prior concepts to engage with this unit: • Tables of equivalent ratios • Representations of sets of equivalent ratios **Relevant Unit(s)/Lesson(s) to Review:** • 6ACC Unit 1 • 6ACC Unit 2, Lesson 9, Activity 2 • 6 ACC Unit 2 Lesson 18	This unit builds on previous work with equivalent ratios and prepares students for the study of linear functions in later courses. It also applies to measuring circles and later to the volume of spheres, cylinders, and cones
Differentiation through *Universal Design for Learning*	

Engagement:
- Provide calculators to facilitate information processing (Lesson 6, Activity 2 Launch)
  LT2: Write an equation of the form y=kx to represent a proportional relationship

Representation:
- Use physical snap cubes to connect symbols to concrete objects like "side length" (Lesson 5, Activity 2 Launch)
  LT1: Use an equation to solve problems involving a proportional relationship

Action & Expression:
- Chunk the task of drawing a classroom floor plan into manageable parts (Lesson 13, Activity 3 Launch)
  LT9: Solve problems involving circumference

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Displa*y - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:
- MLR5: Co-Craft Questions (Lessons 2, 3, 5, 7, 10, 11, 13, 14, 16, 19)
- MLR7: Compare and Connect (Lessons 1, 5, 8, 9, 13, 14, 18, 19, 20)
- MLR8: Discussion Supports (Lessons 1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare
- Approaches to solving problems involving proportional relationships (Lesson 3).
- Proportional relationships with nonproportional relationships (Lesson 5).
- Tables, descriptions, and graphs representing the same situations (Lesson 7).
- Graphs of proportional relationships (Lesson 8).
- The relationships of square diagonals and perimeters to square diagonals and areas (Lesson 10).
- The relationships of diameters and circumferences to diameters and areas (Lesson 15).

Justify
- Reasoning about circumference and perimeter (Lesson 13).
- Estimates for the areas of circles (Lesson 15).
- Reasoning about areas of curved figures (Lesson 16).
- Whether or not a relationship is proportional (Lesson 17).
- Reasoning about the cost of stained-glass windows (Lesson 20).

Generalize
- About proportional relationships (Lesson 1).
- About equations that represent proportional relationships (Lesson 2).
- About categories for sorting circles (Lesson 11).
- About the relationships between circumference and diameter (Lesson 12).

Sentence Frames and Stems

Section A
- The equation _____ represents this proportional relationship because ...
- Two equations that represent the same proportional relationship are _____ and _____.
- I used _____ to represent the relationship and find the unknown values to be ...
- I used the equation _____ to find the number of _____ when the number of _____ is _____.

Section B
- This situation is/is not a proportional relationship because ...
- The values in this table do/do not represent a proportional relationship because ...
- The equation _____ represents a proportional relationship because ...

Section C

- The graph does/does not represent a proportional relationship because ...
- Using the graph, I found the constant of proportionality to be _____ because ...
- The constant of proportionality of this graph describes the relationship between _____ and _____.
- The coordinates _____ represent ...
- The constant of proportionality of _____ is greater/less than the constant of proportionality of _____ because ...
- The equation _____ represents the proportional relationship shown in the graph because ...

Section D

- The diameter of the circle is _____.
- I know the radius of the circle is _____ because the diameter is _____ and ...
- The equation _____ can be used to find the circumference of a circle with a diameter/radius of _____.
- The diameter of the circle is _____ with a circumference of _____. I know these values have a proportional relationship because ...
- If I know the _____ of a circle, I can find the _____ by ...
- The circumference of the circle is _____ because ...

Section E

- The difference between circumference and area of a circle is ...
- I calculated the area of the circle by ...
- The area of the circle is _____ because ...
- The equation _____ can be used to find the area of a circle with a diameter/radius of _____.
- To find the area of a shaded region, first I _____, then I _____ ...

Section F

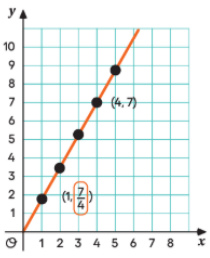
- I created a proportional relationship between _____ and _____. The constant of proportionality is _____ and an equation to represent the relationship is _____.
- I am most comfortable representing a proportional relationship with _____ because ...
- For this question, I need to find the _____ of a circle because...
- I know the _____ of the circle, which I can use to find the _____. This will help me solve this problem because ...
- I used the equation _____ to solve this problem because ...

Unit Outline

In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on previous work with equivalent ratios and helps prepare students for the study of linear functions in later courses.

In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a “proportional relationship,” and the unit rate that describes this relationship is called a “constant of proportionality.” Students use equations of the form $y = kx$ to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship.

Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through (0,0). They interpret points on the graph, including the point (1,k). Here is an example of a graph, an equation, and a table that all represent the same proportional relationship.



$y = \frac{7}{4}x$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

Next, students apply their knowledge of proportional relationships to the context of measuring circles. This builds on students’ work from previous grades with perimeter and area of polygons. Students will build on this work in later courses when they study the volume of spheres, cylinders, and cones. The terms “center,” “radius,” “diameter,” and “circumference” are introduced. Then students investigate the relationship between circumference and diameter and see that it is a proportional relationship. They apply this relationship to solve problems. Next, students explore the area of circular regions. They see an informal derivation that shows where the formula $A=\pi r^2$ comes from and then use this formula to solve problems.

A note on using the terms "ratio," "proportional relationship," and "unit rate":
In these materials, the term "ratio" is used to mean a type of association between two or more quantities. A quantity is a measurement that can be specified by a number and a unit, for example 4 oranges, 4 centimeters, or "my height in feet." A proportional relationship is a collection of equivalent ratios.

A unit rate is the numerical part of a rate per 1 unit, for example, the 6 in 6 miles per hour. The fractions a/b and b/a are never called ratios. The fractions a/b and b/a are identified as "unit rates" for the ratio a:b.

In high school—after the study of ratios, rates, and proportional relationships—students discard the term "unit rate" and start referring to a *to* b, $a:b$, *and* a/b *as "ratios."*

In Accelerated 6 and 7, students write rates without abbreviated units, for example as "3 miles per hour" or "3 miles in every 1 hour." Use of notation for derived units such as mi/hr waits for high school—except for the special cases of area and volume.

A note on using the term "circle":
Strictly speaking, a circle is one-dimensional. It is the boundary of a two-dimensional region, rather than the region itself. The circular region is called a "disk." Because students are not yet expected to make this distinction, these materials refer to both disks and the boundaries of disks as "circles," using illustrations to eliminate ambiguity.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Representing Proportional Relationships with Equations (Lessons 1-3)	**Learning Target #1** Use an equation to solve problems involving a proportional relationship. **Learning Target #2** Write an equation of the form y=kx to represent a proportional relationship, given a table or a description of the situation.	**Lesson 1 Proportional Relationships and Equations** • I can write an equation of the form y=kx to represent a proportional relationship shown in a table or described in a story. • I can write the constant of proportionality as an entry in a table. **Lesson 2 Two Equations for Each Relationship** • I can find two constants of proportionality for a proportional relationship. • I can write two equations representing a proportional relationship described by a table or story. **Lesson 3 Writing Equations to Represent Relationships** • I can find missing information in a proportional relationship using the constant of proportionality. • I can relate all parts of an equation like y=kx to the situation it represents.
Checkpoint A	***Responding to Student Thinking*** • **Problem 1:** Points to Emphasize: If most students struggle with writing an equation that represents a proportional relationship, as opportunities arise over the next several lessons, revisit the structure and meaning of an equation of the form y=kx. For example, in the activity referred to here, invite multiple students to share their thinking about the structure of the equations. ○ Accelerated 6, Unit 5, Lesson 5, Activity 2 Total Edge Length, Surface Area, and Volume • **Problem 2:** Points to Emphasize: If most students struggle with using an equation to find unknown values, revisit this concept as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about substituting values and evaluating expressions. • Accelerated 6, Unit 5, Lesson 5, Activity 1 More Conversions	
Section B Comparing Proportional and Nonproportional Relationships (Lessons 4-6)	**Learning Target #3** Determine whether the values in a table could represent a proportional relationship **Learning Target #4** Use a table to determine whether an equation represents a proportional relationship.	**Lesson 4 Comparing Relationships with Tables** • I can decide if a relationship represented by a table could be proportional and when it is definitely not proportional. **Lesson 5 Comparing Relationships with Equations** • I can decide if a relationship represented by an equation is proportional or not. **Lesson 6 Solving Problems about Proportional Relationships** • I can ask questions about a situation to determine whether two quantities are in a proportional relationship. • I can solve all kinds of problems involving proportional relationships.
Checkpoint B	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of determining whether or not a table could represent a proportional relationship. If students struggle with this, make time to examine related work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier work. ○ Accelerated 6, Unit 5, Section B Comparing Proportional and Nonproportional Relationships • **Problem 2:** More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.	
Section C Representing Proportional Relationships with	**Learning Target #5** Determine whether a given graph represents a proportional relationship.	**Lesson 7 Graphs of Proportional Relationships** • I can find the constant of proportionality from a graph. • I know that the graph of a proportional relationship lies on a line through (0,0)

Graphs (Lessons 7-9)	**Learning Target #6** Identify the constant of proportionality from the graph of a proportional relationship. **Learning Target #7** Interpret points on the graph of a proportional relationship.	• I can write an equation representing a proportional relationship from a graph. **Lesson 8 Using Graphs to Compare Relationships** • I can compare two, related proportional relationships based on their graphs. • I know that the steeper graph of two proportional relationships has a larger constant of proportionality. **Lesson 9 Two Graphs for Each Relationship** • I can interpret a graph of a proportional relationship using the situation.
Checkpoint C	***Responding to Student Thinking*** • **Problem 1:** Points to Emphasize: If most students struggle with determining whether or not a graph represents a proportional relationship, revisit this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about this using the graphs in the lessons referred to here. ○ Accelerated 6, Unit 5, Lesson 10 How Well Can You Measure ○ Accelerated 6, Unit 5, Lesson 12 Exploring Circumference • **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of interpreting points on the graph of a proportional relationship. If students struggle with this, make time to examine related work in the lessons referred to here. The Course Guide provides additional ideas for revisiting earlier work. ○ Accelerated 6, Unit 5, Lesson 7 Interpreting Graphs of Proportional Relationships ○ Accelerated 6, Unit 5, Lesson 9 Two Graphs for Each Relationship	
Section D Circumference of a Circle (Lessons 10-13)	**Learning Target #8** Recognize that there are proportional relationships between the circumference, diameter, and radius of circles and express these relationships using equations. **Learning Target #9** Solve problems involving circumference.	**Lesson 10 How Well Can You Measure?** • I can examine quotients and use a graph to decide whether two associated quantities are in a proportional relationship. • I understand that it can be difficult to measure the quantities in a proportional relationship accurately. **Lesson 11 Exploring Circles** • I can describe the characteristics that make a shape a circle. • I can identify the diameter, center, radius, and circumference of a circle. **Lesson 12 Exploring Circumference** • I can describe the relationship between circumference and diameter of any circle. • I can explain what π means. **Lesson 13 Applying Circumference** • I can choose an approximation for π based on the situation or problem. • If I know the radius, diameter, or circumference of a circle, I can find the other two.
Checkpoint D	***Responding to Student Thinking*** • **Problem 1:** More chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. • **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of finding the circumference of a circle given its radius. If students struggle with this, make time to examine related work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier work. ○ Accelerated 6, Unit 5, Section D Circumference of a Circle	
Section E Area of a Circle (Lessons 14-16)	**Learning Target #10** Justify that the area of a circle can be calculated with the formula A=pir^2 **Learning Target #11** Recognize that the area of a circle is not proportional to its diameter or radius. **Learning Target #12** Solve problems involving the area of a circle.	**Lesson 14 Estimating Areas** • I can calculate the area of a complicated shape by breaking it into shapes whose area I know how to calculate. **Lesson 15 Area of a Circle** • I know the formula for the area of a circle. • I know whether or not the relationship between the diameter and area of a circle is proportional and can explain how I know. **Lesson 16 Applying Area of Circles** • I can calculate the area of more complicated shapes that include fractions of circles.

		• I can write exact answers in terms of π
Checkpoint E	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of recognizing that the area of a circle is not proportional to the diameter. If students struggle with this, make time to examine related work in the lessons referred to here. The Course Guide provides additional ideas for revisiting earlier work. ○ Accelerated 6, Unit 5, Lesson 15 Area of a Circle • **Problem 2:** Points to Emphasize: If most students struggle with finding the area of a circle and expressing it in terms of π, revisit this concept when opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the situations involving area in this activity: ○ Accelerated 6, Unit 5, Lesson 19, Activity 1 Card Sort: Circle Problems	
Section F Let's Put it To Work (Lesson 17-20)	**No new learning targets**	**Lesson 17 Four Representations** • I can make connections between the graphs, tables, and equations of a proportional relationship. • I can use units to help me understand information about proportional relationships. **Lesson 18 Using Water Efficiently** • I can answer a question by representing a situation using proportional relationships. **Lesson 19 Distinguishing Circumference and Area** • I can decide whether a situation about a circle has to do with area or circumference. • I can use formulas for circumference and area of a circle to solve problems. **Lesson 20 Stained-Glass Windows** • I can apply my understanding of area and circumference of circles to solve more complicated problems.
End of Unit Assessment		

Unit Title:

Unit 2: Percent Increase and Decrease (G6 ACC U6)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	7.RP.A.1, 7.RP.A.2	Lesson 8	7.EE.A.2, 7.RP.A.3
Lesson 2	7.EE.A.2, 7.RP.A.2	Lesson 9	7.RP.A.3
Lesson 3	7.NS.A.2.d, 7.RP.A.2	Lesson 10	7.RP.A.3
Lesson 4	7.RP.A.3	Lesson 11	7.RP.A.3
Lesson 5	7.RP.A.3	Lesson 12	7.RP.A.3
Lesson 6	7.EE.A.2, 7.RP.A.3	Lesson 13	7.RP.A.3
Lesson 7	7.RP.A.3		

Essential Question(s):

- How can the distributive property simplify multi-step percent problems?
- What is the significance of "100%" when modeling changes in value?
- How do we quantify the accuracy of a measurement using percentages?

Enduring Understanding(s):

- Percent increase and decrease can be represented as a single multiplication operation (e.g., 1.15x for a 15% increase) by applying the distributive property to the expression x+0.15x
- In any situation involving percent increase or decrease, the original amount always corresponds to 100% on a double number line or tape diagram
- Percent error is a measurement of how far off an estimate is from the actual value, expressed as a percentage of that actual value

Demonstration of Learning:

CFA1: Checkpoint A (after lesson 3)
CFA 2: Checkpoint B (after lesson 7)
CFA 3: Checkpoint C (after lesson 11)
EoU: Assessment A (after lesson 11)

Pacing for Unit

17 Days
Lesson Modifications:

- Covering Lessons 1-11
- Optional Lessons: 7, 12, 13
- 2 days for review/assess
- 4 flex days for reteaching of prior percent content (Acc 6th unit 2, lessons 21-26 suggested for teachers to take a look at)

Family Overview

https://accessim.org/6-8-accelerated/accelerated-6/unit-6?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
Acc6.6.2	(a fraction) more than (a fraction) less than initial / original amount final / new amount	tape diagram distributive property
Acc6.6.3	repeating decimal decimal representation	
Acc6.6.4	percent increase percent decrease	(a fraction) more than (a fraction) less than
Acc6.6.5	discount	initial / original amount final / new amount
Acc6.6.8	sales tax tax rate tip	percent increase
Acc6.6.9	interest commission	percent decrease

Aligned Unit Materials, Resources, and Technology

Provide access as needed throughout the unit:

- Four-function calculators
- Grocery store circulars (Grocery store advertisements from the newspaper or that are picked up at the store. If students have internet access, an online version could be substituted.)
- Math Community Chart
- Sticky notes
- Tools for creating a visual display

Lesson	Materials to Gather	Materials to Copy
2		Fractional Relationship Cards (1 copy for every 2 students): Activity 2
3		More Representations Cards (1 copy for every 2 students): Activity 3
6	Four-function calculators: Activity 1	

	markup markdown	
Acc6.6.10		discount
Acc6.6.11	measurement error percent error	

8	Four-function calculators: Activity 1, Activity 2, Activity 3	
9	• Math Community Chart • Four-function calculators: Activity 2	
10	Four-function calculators: Activity 1	
11	Four-function calculators: Activity 3	
13	• Grocery store circulars: Warm-up, Activity 1 • Sticky notes: Activity 2 • Tools for creating a visual display: Activity 2	

Opportunities for Interdisciplinary Connections:

- **Culinary Arts:** Students apply proportional reasoning to scale food and drink recipes
- **Environmental Science:** The unit explores water usage efficiency in households and the economic value of recycling aluminum cans

Anticipated misconceptions:

Additive Discounts: Students may think that two successive 25% discounts are the same as one 50% discount

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

Essential prior concepts to engage with this unit:

- Use proportional relationships to find equivalent ratios
- Ratios and unit rates
- Solving percent problems

Relevant Unit(s)/Lesson(s) to Review:

- Acc Unit 2 Lessons 21-24
- Grade 7, Unit 4, Lesson 2

Connections to Future Units:

Groundwork for Grade 9/high school work with exponential functions representing compounded percent changes. It also prepares students for real-world applications like tax, tip, simple interest, markup, and markdown.

Differentiation through *Universal Design for Learning*

Engagement:

- Provide rubrics or checklists for creating visual displays for news reports (Lesson 13, Activity 2 Student Task Statement)
  LT3: Create algebraic expressions or equations that represent a situation involving percent increase or decrease

Representation:

- Use a double number line diagram to help organize information (Lesson 5, Activity 2 Launch)
  LT3: Create algebraic expressions or equations that represent a situation involving percent increase or decrease

Action & Expression:

- Invite students to demonstrate measurement ideas using a physical ruler (Lesson 11, Activity 1 Student Task Statement)
  LT4: Calculate measurement error and express it as a percentage

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 1, 6, 11, 12)
- MLR6: Three Reads (Lessons 5, 11, 12)
- MLR8: Discussion Supports (Lessons 2, 3, 4, 6, 7, 8, 9, 11, 13)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and representing. Throughout the unit, students will benefit from routines designed to grow robust

disciplinary language, both for their own sensemaking and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Concrete problems involving percent increase and decrease (Lesson 5).
- Problems involving sales tax and tip (Lesson 8).
- Concrete situations involving percent error (Lesson 11).

Explain

- How to solve concrete and abstract problems involving an amount plus (or minus) a fraction of that amount (Lesson 2).
- How to solve percent change problems (Lesson 4).
- Strategies for solving percent problems with fractional percentages (Lesson 7).
- How to measure lengths and interpret measurement error (Lesson 11).
- Strategies for solving percent error problems (Lesson 11).

Represent

- Situations involving percent increase and decrease (Lessons 6 and 12).
- Situations from the news involving percent change (Lesson 13).

Sentence Frames and Stems

Section A

- I know _____ is/is not a scaled copy of _____ because ...
- The scale factor from _____ to _____ is _____ because ...
- The constant of proportionality is ____, and I can use it to help me find _____ because ...
- I can use the distributive property to represent _____ more than _____ with the equation _____.
- I used long division to generate the decimal _____ from the fraction _____.

Section B

- The amount increased/decreased by _____ which is _____ percent of the original amount _____.
- I used _____ to represent _____ percent increase/decrease because ...
- I found the new amount to be _____ after the original amount _____ increased/decreased by _____ percent.
- I wrote the equation_____ to model this situation because ...

Section C

- In this situation, the _____ can be represented with a percent increase/decrease and the equation _____.
- I used the decimal value _____ to represent the percent _____ in the expression _____ to find the _____.
- If the regular price is _____, a discount of _____ percent would make the sale price _____.
- The percent error in this situation is _____ because ...

Section D

- There is a percent increase/decrease of _____ from _____ to _____.
- I know this situation is asking about a percent increase/decrease because ...
- I used _____ to model the situation because ...
- I used the equation _____ to solve this problem because ...

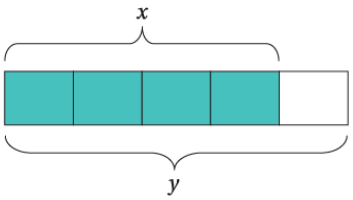
Unit Outline

In this unit, students deepen their understanding of proportional relationships and percentages. They solve multi-step problems and work with situations that involve fractional amounts. This builds on the work students did in previous units with ratios, rates, percentages, and proportional relationships. Students will build on this work in high school with exponential functions representing compounded percent increase and decrease.

Students begin the unit by revisiting proportional relationships, but this time the given values are fractional amounts. To determine the constant of proportionality, students must compute the quotient of two fractions. Students also make sense of situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and apply the distributive property to generate expressions that represent these situations. They also use long division to write fractions as decimals, including their first introduction to repeating decimals.

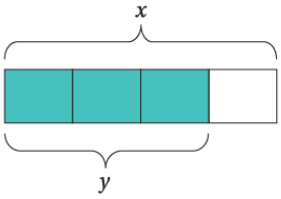
Next, students make sense of situations where an increase or decrease is expressed as a percentage of the initial amount. They continue creating diagrams and writing equations to represent the situations. They solve for any one of the three quantities—the initial amount, the final amount, or the percentage of the change—given the other two quantities. They also reason about fractional percentages.

Then students apply percent increase and decrease to solve problems in a variety of real-world situations, such as tax, tip, interest, markup, discount, depreciation, and commission. Lastly, students make sense of situations where the difference between a correct measurement and an incorrect measurement is expressed as a percentage of the correct amount.



$y = x + \frac{1}{4}x$
$y = (1 + \frac{1}{4})x$
$y = \frac{5}{4}x$

$y = (1 + 0.25)x$
$y = 1.25x$
"a 25% increase"



$y = x - \frac{1}{4}x$
$y = (1 - \frac{1}{4})x$
$y = \frac{3}{4}x$

$y = (1 - 0.25)x$
$y = 0.75x$
"a 25% decrease"

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Revisiting Proportional Relationships (Lessons 1-3)	**Learning Target #1** Create algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value. **Learning Target #2** Solve problems about proportional relationships with fractional quantities. Use long division to generate a decimal representation of a fraction.	**Lesson 1 Revisiting Proportional Relationships** • I can use a table with 2 rows and 2 columns to find an unknown value in a proportional relationship. • When there is a constant rate, I can identify the two quantities that are in a proportional relationship. **Lesson 2 Percent Increase and Decrease** • I can use the distributive property to rewrite an expression like x+12x as (1+12)x • I understand that "half as much again" and "multiply by 3/2" mean the same thing. **Lesson 3 Applying Percentages** • I can use the distributive property to rewrite an equation like x+0.5x=1.5x • I can write fractions as decimals. • I understand that "half as much again" and "multiply by 1.5" mean the same thing.

Checkpoint A	***Responding to Student Thinking***
	• **Problem 1:** More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.
	• **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of using the distributive property to generate expressions that represent a fractional increase. If students struggle, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.
	  ○ Accelerated 6, Unit 6, Lesson 2 More than That, Less than That
	• **Problem 3:** Points to Emphasize: If students struggle with converting a fraction to a decimal, revisit this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about finding the percent error in this activity:
	  ○ Accelerated 6, Unit 6, Lesson 11, Activity 2 Plants, Bicycles, and Crowds	

Section B Percent Increase and Decrease (Lessons 4-7)	**Learning Target #3** Create algebraic expressions or equations that represent a situation involving percent increase or decrease. **Learning Target #4** Use diagrams to solve problems involving percent increase or decrease.	**Lesson 4 Increasing and Decreasing** • I can draw a tape diagram that represents a percent increase or decrease. • When I know the starting amount and the percent increase or decrease, I can find the new amount. **Lesson 5 One Hundred Percent** • I can use a double number line diagram to help me solve percent increase and decrease problems. • I understand that if I know how much a quantity has grown, then the original amount represents 100%. • When I know the new amount and the percentage of increase or decrease, I can find the original amount. **Lesson 6 Percent Increase and Decrease with Equations** • I can solve percent increase and decrease problems by writing an equation to represent the situation and solving it. **Lesson 7 Part of a Percent** • I can find fractional percentages (like 12.5% or 0.4%) of quantities. • I understand that to find 0.1% of a quantity, I have to multiply by 0.001.
Checkpoint B	***Responding to Student Thinking*** • **Problem 1:** Points to Emphasize: If students struggle with finding the percentage of an increase or decrease, revisit this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about finding the percentages in this activity: ○ Accelerated 6, Unit 6, Lesson 8, Activity 3 Dining at a Restaurant • **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of representing situations involving percent increase or decrease. If students struggle, make time to	

<table>
<tr><td></td><td colspan="2">revisit related work in the activities referred to here. See the Course Guide for ideas to help students re-engage with earlier work.
○ Accelerated 6, Unit 6, Lesson 4, Activity 2 Using Tape Diagrams
○ Accelerated 6, Unit 6, Lesson 6, Activity 1 Matching Equations</td></tr>
<tr><td>Section C
Applying Percentages
(Lessons 8-11)</td><td>Learning Target #5
Calculate measurement error, and express it as a percentage of the actual value.

Learning Target #6
Solve problems involving tax, tip, simple interest, markup, markdown, or commission.</td><td>Lesson 8 Tax and Tip
● I understand and can solve problems about sales tax and tip.
Lesson 9 Percentage Contexts
● I understand and can solve problems about commission, interest, markups, and discounts.
Lesson 10 Solving Multi-Step Percentage Problems
● I can solve problems that involve multiple percentages.
Lesson 11 Expressing Error as a Percentage
● I can represent measurement error as a percentage of the correct measurement.
● I can solve problems that involve percent error.</td></tr>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
● Problem 1: Press Pause: By this point in the unit, there should be some student mastery of finding the value after tax, tip, markup, or interest is added. If students struggle, make time to revisit related work in this section. See the Course Guide for ideas to help students re-engage with earlier work.
○ Accelerated 6, Unit 6, Section C Applying Percentages
● Problem 2: More Chances: Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time.</td></tr>
<tr><td>Section D
Let's Put it to Work
(Lessons 12-13)</td><td>No new learning targets</td><td>Lesson 12 Changes on the Earth
● I can use percentages to describe changes in real-world situations.
Lesson 13 Posing Percentage Problems
● I can write and solve problems about real-world situations that involve percent increase and decrease.</td></tr>
<tr><td colspan="3">End of Unit Assessment</td></tr>
</table>

Unit Title:

Unit 3: Rational Numbers (G6 ACC U7)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	6.NS.C.5, 6.NS.C.6, 7.NS.A.1	Lesson 13	6.NS.C.6, 6.NS.C.6.b, 6.NS.C.6.c, 6.NS.C.8, 7.NS.A.1.c
Lesson 2	6.NS.C.7.a, 6.NS.C.7.b	Lesson 14	7.NS.A.2.a, 7.NS.A.2.c, 7.RP.A
Lesson 3	6.NS.C, 6.NS.C.5, 6.NS.C.6, 6.NS.C.6.a, 6.NS.C.7	Lesson 15	7.NS.A.2.c
Lesson 4	6.NS.C.7, 6.NS.C.7.c, 6.NS.C.7.d	Lesson 16	7.NS.A.2.b
Lesson 5	6.NS.C.6, 6.NS.C.7, 6.NS.C.7.a, 6.NS.C.7.d	Lesson 17	7.EE.B.3, 7.NS.A.2.a, 7.NS.A.3, 7.RP.A
Lesson 6	7.NS.A.1.a, 7.NS.A.1.b	Lesson 18	7.NS.A, 7.NS.A.1.d, 7.NS.A.2.c
Lesson 7	7.NS.A.1.a, 7.NS.A.1.b, 7.NS.A.1.d	Lesson 19	7.NS.A.3
Lesson 8	7.NS.A.1, 7.NS.A.1.a	Lesson 20	7.EE.B.3, 7.EE.B.4, 7.NS.A, 7.NS.A.3
Lesson 9	7.NS.A.1.c	Lesson 21	7.EE.B.4, 7.NS.A.3
Lesson 10	7.NS.A.1.c, 7.NS.A.1.d, 7.NS.A.3	Lesson 22	6.G.A.3, 6.NS.C.8
Lesson 11	6.NS.C.6.b, 6.NS.C.6.c	Lesson 23	7.EE.B.3, 7.NS.A.3, 7.RP.A.3
Lesson 12	6.NS.C.6.c, 6.NS.C.7.c, 6.NS.C.8		

Essential Question(s):

- How do negative numbers and absolute value describe position and distance?
- What is the relationship between addition and subtraction of rational numbers?
- How are the signs of products and quotients determined when multiplying or dividing rational numbers?

Enduring Understanding(s):

- Negative numbers represent values relative to a reference point of zero, while absolute value represents the distance of a number from zero regardless of its direction.
- Subtracting a rational number is equivalent to adding its additive inverse (the opposite of the number)
- Multiplying or dividing numbers with the same sign results in a positive value, while numbers with different signs result in a negative value

Demonstration of Learning:

CFA 1: Checkpoint B (after lesson 10)
CFA 2: Checkpoint D (after lesson 17)
CFA 3: Checkpoint E (after lesson 21)
EoU: Assessment A (after lesson 21)

- NOTE: Combine mid unit and end of unit (take away 2 coordinate plane questions and add #4 and #6 from mid unit to replace those)

Pacing for Unit

20 Days
Lesson Modifications:

- Optional lessons: 4 (Lessons 17, 21, 22, and 23)
- Omit Lessons 1-5 (absolute value, ordering of #s)
- Omit Lessons 11-13 (coordinate plane)
- Covering Lessons 6-10, 14-21
- 5 flex days

Family Overview

https://accessim.org/6-8-accelerated/accelerated-6/unit-7?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
Acc6.7.1	positive number negative number temperature degrees Celsius elevation sea level closer to 0 farther from 0	number line below zero
Acc6.7.2	rational number sign	greater than less than

Aligned Unit Materials, Resources, and Technology

Digital Applets:

- 7.1 High Places, Low Places
- 7.4 Jumping Bug
- 7.6 Water Temperatures
- 7.7 Cliffs and Caves
- 7.13 Coordinate Patterns, Signs of Numbers in Coordinates, Funding Distances on a Coordinate Plane, Plotting Polygons
- 7.15 Rational Numbers Multiplication Grid
- 7.16 Drilling Down
- 7.18 Seagulls and Sharks Again
- 7.20 Match Solutions
- 7.21 Cat Pictures, Design Your Own Image

	inequality	
Acc6.7.3	opposite (numbers) from least to greatest	
Acc6.7.4	absolute value	positive number negative number distance (away) from 0
Acc6.7.5		closer to 0 farther from 0
Acc6.7.6	signed numbers	temperature
Acc6.7.7	sum expression	
Acc6.7.8	deposit withdrawal account balance debt	
Acc6.7.10	difference	absolute value distance
Acc6.7.11	quadrant coordinate plane x-coordinate y-coordinate (line) segment	axis
Acc6.7.12	degrees Fahrenheit	degrees Celsius
Acc6.7.13		absolute value x-coordinate y-coordinate
Acc6.7.14	velocity	
Acc6.7.16	solution (to an equation) factor	
Acc6.7.18	additive inverse multiplicative inverse rational number variable	sum difference
Acc6.7.20		opposite solution (to an equation)
Acc6.7.23		increase decrease

Provide access as needed throughout the unit:

- Four-function calculators
- Graphing technology
  Examples of graphing technology are a handheld graphing calculator, a computer with a graphing calculator application installed, and an internet-enabled device with access to a site like desmos.com/calculator. For students using the digital materials, a separate graphing calculator tool isn't necessary because interactive applets are embedded throughout, and a graphing calculator tool is accessible on the student digital toolkit page.
- Graph paper
- Math Community Chart
- Receipt tape
- Tools for creating a visual display

Lesson	Materials to Gather	Materials to Copy
3		Ordering Rational Numbers Cards (1 copy for every 2 students): Activity 1
5		Points on the Number Line Cards (1 copy for every 4 students): Activity 1
7	Receipt tape: Activity 3	
10	Math Community Chart: Activity 2	
15		• Rational Numbers Multiplication Grid Handout (1 copy for each student): Activity 1 • Matching Expressions Cards (1 copy for every 2 students): Activity 2
18		The Same but Different Cards (1 copy for every 2 students): Activity 1
20		Matching Inverses Cards (1 copy for every 2 students): Activity 3
21	Tools for creating a visual display: Activity 3	
22	• Graph paper: Warm-up, Activity 1 • Graphing technology: Warm-up, Activity 1	
23	Four-function calculators: Activity 1, Activity 2, Activity 3	Your Own Stock Portfolio Handout (1 copy for each student): Activity 3

Opportunities for Interdisciplinary Connections:

Geography: Students use absolute value and signed numbers to describe distances and locations relative to sea level

Anticipated misconceptions:

Absolute Value Confusion: Students might treat absolute value as the opposite of a number rather than its distance from zero

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

Essential prior concepts to engage with this unit:

- Whole numbers and non-negative fractions (Grades K–4)

Connections to Future Units:

Serves as a natural lead-in to solving equations and simplifying expressions in Unit 5 of this course. It also facilitates the transition to plotting coordinate pairs in all four quadrants of the

• Understanding of fractions, decimals, (Grades 4-5) • Ordering/comparing numbers from (Grade 5) • First-quadrant coordinate graphing (Grade 5) • Multiplication and division of whole numbers and fractions (Grades 3–6) **Relevant Unit(s)/Lesson(s) to Review:** • Truth and Equations (Acc6.4.2) • Staying in Balance (Acc6.4.3) • Practice Solving Equations (Acc6.4.4) • Equivalent Expressions (Acc6.4.7)	coordinate plane

Differentiation through *Universal Design for Learning*

Engagement:
- Leverage choice around perceived challenge by inviting students to select specific polygons to plot (Lesson 13, Activity 3 Student Task Statement)
  LT7: Use coordinates and absolute value to find distances between points

Representation:
- Maintain a visible vocabulary display for terms like "absolute value" with pictures (Lesson 4, Activity 2 Activity Synthesis)
  LT1: Interpret a rational number and the absolute value of a number in context

Action & Expression:
- Provide students with access to blank number lines from -10 to 30 to support representing transactions (Lesson 8, Activity 1 Student Task Statement)
  LT4: Apply addition and subtraction of signed numbers to represent situations

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:
- MLR2: Collect and Display (Lessons 3, 7, 8, 13, 18, 20)
- MLR7: Compare and Connect (Lessons 4, 6, 9, 13, 16, 20, 21)
- MLR8: Discussion Supports (Lessons 1, 4, 6, 10, 11, 14, 15, 18, 19, 20, 21)

Progression of Interdisciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, representing, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sensemaking and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret
- Situations involving negative numbers (Lessons 1 and 5).
- Graphs involving positive and negative numbers (Lesson 12).
- Tables and situations involving signed numbers (throughout unit).

Represent
- Addition of signed numbers on a number line (Lesson 6).
- Situations involving signed numbers (Lessons 7, 10, and 16).
- Changes in elevation (Lesson 10).
- Position, speed, and direction (Lesson 14).

Generalize
- About subtracting and adding signed numbers (Lesson 9).
- About differences and magnitude (Lesson 10).
- About multiplying negative numbers (Lessons 14 and 15).
- About additive and multiplicative inverses (Lesson 20).

Sentence Frames and Stems

Section A

- _____ is greater/less than _____ because ...
- The opposite of _____ is _____ because ...
- The absolute value of _____ is _____.
- The value _____ is located _____ spaces to the right/left of zero on a number line which means its opposite, _____ is located _____ spaces to the right/left of zero.
- The value _____ makes sense in this situation because ...
- To add/subtract _____ and/from _____ on the number line, first I _____ then I...
- The equation _____ represents this situation because ...
- The sum of _____ and _____ is _____ because ...
- The difference between _____ and _____ is _____ because ...
- I used the number _____ to represent this situation because ...

Section B

- To add/subtract _____ and/from _____ on the number line, first I _____ then I...
- The equation _____ represents this situation because ...
- The sum of _____ and _____ is _____ because ...
- The difference between _____ and _____ is _____ because ...
- I used the number _____ to represent this situation because ...

Section C

- Point _____ is in quadrant _____ because ...
- The distance between the point _____ and the point _____ is _____ because ...
- To plot the point _____, first I _____, then I _____ and place the point.
- The expressions _____ and _____ are equivalent because ...
- To find the difference between _____ and _____, I can ...
- _____ is the multiplicative inverse of _____ because ...
- The equation _____ fits this situation, where the variable _____ represents ...
- The solution to the equation _____ is _____ because ...

Section D

- The product/quotient of _____ and _____ is _____ because ...
- The equation _____ fits this situation because ...
- _____ and _____ are opposites because ...
- The value of the expression _____ will be positive/negative because ...

Section E

- The expressions _____ and _____ are equivalent because ...
- To find the difference between _____ and _____, I can ...
- _____ is the multiplicative inverse of _____ because ...
- The equation _____ fits this situation, where the variable _____ represents ...
- The solution to the equation _____ is _____ because ...

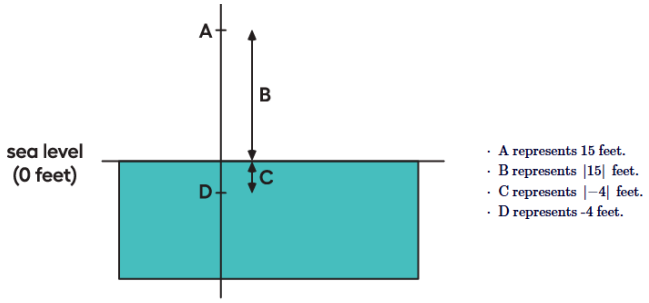
Section F

- I chose to design a _____ on the coordinate plane and used the ordered pairs _____.
- I began my design in the _____ quadrant because ...
- The equation _____ can be used to find the missing value _____ for company _____ .
- The company saw their shares increase/decrease by _____ percent.

Unit Outline

In this unit, students learn about negative numbers and ways to represent them on a number line and the coordinate plane. They perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction or zero.

Students begin by considering situations involving temperature or elevation and interpreting what negative numbers mean in those contexts. Previously, when students worked only with nonnegative numbers, magnitude and order were indistinguishable. In this unit, when comparing two signed numbers, students learn to distinguish between the absolute value of a number (magnitude) and a number's relative position on the number line (order).



Then students use tables and number line diagrams to represent changes in temperature or elevation. They extend addition and subtraction from fractions to all rational numbers. And they see that $a - b$ is equivalent to $a + (-b)$.

Then students use ordered pairs to describe pairs of numbers that include negative numbers. In Grade 5, they plotted pairs of positive numbers on the coordinate grid. Here, they plot pairs of rational numbers in all four quadrants of the coordinate plane. They interpret the meanings of plotted points in given contexts and use coordinates to calculate horizontal or vertical distances between two points.

Next, students examine multiplication and division. They work with constant velocity, which is a signed number that indicates direction and speed. This allows products of signed numbers to be interpreted in terms of position, direction of movement, and time before or after a specific point. Students use the relationship between multiplication and division to understand how division extends to rational numbers.

Then students work with expressions that use the four operations on rational numbers. They also solve problems that involve interpreting negative numbers in context. They solve linear equations of the form $x+p=q$ or $px=q$, where p and q are rational numbers.

A note on using the terms "expression," "equation," and "signed number":
In these materials, an expression is built from numbers, variables, operation symbols ($+,-,\cdot,\div$), parentheses, and exponents. (exponents—in particular, negative exponents—are not a focus of this unit. Students work with integer exponents in a future course and noninteger exponents in high school.) An equation is a statement that two expressions are equal, thus it always has an equal sign. Signed numbers include all rational numbers, written as decimals or in the form $\frac{a}{b}$.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Negative Numbers and Absolute Value (Lessons 1-5)	**Learning Target #1** Interpret a rational number and the absolute value of a number in context. **Learning Target #2** Plot rational numbers and their opposites on a number line; know that a number and its opposite have the same absolute value. **Learning Target #3** Use words and symbols to compare rational numbers, where a rational number could also be the absolute value of a number.	**Lesson 1 Positive and Negative Numbers** • I can explain what 0, positive numbers and negative numbers mean in the context of temperature and elevation. • I can use positive and negative numbers to describe temperature and elevation. • I know what positive and negative numbers are. **Lesson 2 Comparing Positive and Negative Numbers** • I can explain how to use the positions of numbers on a number line to compare them. • I can use inequalities to compare positive and negative numbers. **Lesson 3 Ordering Rational Numbers** • I can compare and order rational numbers. • I can use phrases like "greater than," "less than," and "opposite" to compare rational numbers. **Lesson 4 Absolute Value of Numbers** • I can explain what the absolute value of a number is. • I can find the absolute values of rational numbers. • I can recognize and use the notation for absolute value. **Lesson 5 Comparing Numbers and Distance from Zero** • I can explain what absolute value means in situations involving elevation. • I can use absolute values to describe elevations. • I can use inequalities to compare rational numbers and the absolute values of rational numbers.
Checkpoint A	***Responding to Student Thinking*** • **Problem 1:** Point to Emphasize: If most students struggle with locating points on the number line, including opposites and absolute values, reinforce the idea in the next section. For example, when students notice and wonder about points on the number line in the Warm-up referred to here, ask students to determine possible values for each letter based on its location. ○ Accelerated 6, Unit 7, Lesson 7, Activity 1, That's the Opposite • **Problem 2:** Points to Emphasize: If most students struggle with using inequality signs to compare values, especially absolute values, reinforce the idea in the next section. For example, when finding the distances between points in the coordinate plane in the activity referred to here, relate the idea of finding the distance of each point to the x- or y-axis with absolute value. ○ Accelerated 6, Unit 7, Lesson 13, Activity 1, Signs of Numbers in Coordinates • **Problem 3:** More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.	
Section B Adding and Subtracting Rational Numbers	**Learning Target #4** Apply addition and subtraction of signed numbers to represent	**Lesson 6 Changing Temperatures** • I can use a number line to add positive and negative numbers. **Lesson 7 Changing Elevation** • I can add positive and negative numbers.

(Lessons 6-10)	situations and solve problems. **Learning Target #5** Calculate the sum or difference of two rational numbers.	**Lesson 8 Money and Debts** ● I understand what positive and negative numbers mean in a situation involving money. **Lesson 9 Representing Subtraction** ● I can explain the relationship between addition and subtraction of rational numbers. ● I can use a number line to subtract positive and negative numbers. **Lesson 10 Finding Differences** ● I can subtract positive and negative numbers.
Checkpoint B	***Responding to Student Thinking*** ● **Problem 1:** Points to Emphasize: If most students struggle with adding or subtracting rational numbers, revisit this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the addition and subtraction expressions in this activity: ○ Accelerated 6, Unit 7, Lesson 18, Activity 1 Card Sort: The Same but Different ● **Problem 2:** More Chances: Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time.	
Section C The Coordinate Plane (Lessons 11-13)	**Learning Target #6** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **Learning Target #7** Use coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **Learning Target #8** Working in all four quadrants, plot a point given its coordinates, or identify the coordinates of a given point in the coordinate plane.	**Lesson 11 Constructing the Coordinate Plane** ● I can plot points with negative coordinates in the coordinate plane. ● I know what a coordinate plane is and can describe the four quadrants. ● When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes. **Lesson 12 Interpreting Points in a Coordinate Plane** ● I can explain how rational numbers represent balances in a money context. ● I can explain what points in a four-quadrant coordinate plane represent in a situation. ● I can plot points in a four-quadrant coordinate plane to represent situations and solve problems. **Lesson 13 Distances in the Coordinate Plane** ● I can find horizontal and vertical distances between points on the coordinate plane.
Checkpoint C	***Responding to Student Thinking*** ● **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of plotting and labeling the coordinates of points in the coordinate plane. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work. ○ Accelerated 6, Unit 7, Section C The Coordinate Plane ● **Problem 2:** Point to Emphasize: If most students struggle to recognize that when points are related by reflections, their coordinates only differ by signs, revisit this idea in the next section. For example, in the practice problem referred to here, sketch the possible locations of points C and D on a coordinate plane, and notice how the coordinates are opposites. ○ Accelerated 6, Unit 7, Lesson 15, Practice Problem 6 ● **Problem 3:** Points to Emphasize: If most students struggle to find the distance between two points that both lie on the same vertical or horizontal line, revisit this idea in the next section. For example, ask students to plot the points in the practice problem referred to here, and ask them if they notice any connections between the length of the side and the numbers in the coordinates. ○ Accelerated 6, Unit 7, Lesson 14, Practice Problem 5	
Section D Multiplying and Dividing Rational Numbers (Lessons 14-17)	**Learning Target #9** Apply multiplication and division of signed numbers to represent situations and solve problems.	**Lesson 14 Multiplying Rational Numbers** ● I can explain what it means when time is represented with a negative number in a situation about speed and direction. ● I can use rational numbers to represent speed and direction. **Lesson 15 Multiply!** ● I can solve problems that involve multiplying rational numbers.

	Learning Target #10 Calculate the product or quotient of two rational numbers.	**Lesson 16 Dividing Rational Numbers** ● I can divide rational numbers. ● I can solve problems that involve multiplying and dividing rational numbers. **Lesson 17 Negative Rates** ● I can solve problems that involve negative rates.
Checkpoint D	***Responding to Student Thinking*** ● **Problem 1:** Points to Emphasize: If most students struggle with multiplying or dividing rational numbers, revisit this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the multiplication and division expressions in this activity: ○ Accelerated 6, Unit 7, Lesson 18, Activity 1 Card Sort: The Same but Different ● **Problem 2:** More Chances: Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time.	
Section E Four Operations with Rational Numbers (Lessons 18-21)	**Learning Target #11** Apply the four operations with rational numbers to solve problems. **Learning Target #12** Solve an equation of the form $x+p=q$ or $px=q$, where p, q, and x are rational numbers. **Learning Target #13** Write an equation of the form $x+p=q$ or $px=q$ (where p, q, and x are rational numbers) to represent a situation.	**Lesson 18 Expressions with Rational Numbers** ● I can add, subtract, multiply, and divide rational numbers. ● I can evaluate expressions that involve rational numbers. **Lesson 19 Solving Problems with Rational Numbers** ● I can solve equations that include rational numbers and have rational solutions. **Lesson 20 Solving Equations with Rational Numbers** ● I can explain what the solution to an equation means for the situation. **Lesson 21 Representing Contexts with Equations** ● I can write and solve equations to represent situations that involve rational numbers.
Checkpoint E	***Responding to Student Thinking*** ● **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of solving equations of the form $x+p=q$ or $px=q$. If most students struggle, make time to revisit related work in the activities referred to here. See the Course Guide for ideas to help students re-engage with earlier work. ○ Accelerated 6, Unit 7, Lesson 20, Activity 1 Match Solutions ○ Accelerated 6, Unit 7, Lesson 21, Activity 1 Warmer or Colder Than Before? ● **Problem 2:** More Chances: Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time. ● **Problem 3:** Points to Emphasize: If most students struggle with solving problems that involve the four operations with rational numbers, revisit this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the operations they use to complete the table in this activity: ○ Accelerated 6, Unit 7, Lesson 23, Activity 2 What Is a Stock Portfolio?	
Section F Let's Put it To Work (Lesson 22-23)	**No new learning targets**	**Lesson 22 Drawing in the Coordinate Plane** ● I can use ordered pairs to draw a picture. **Lesson 23 The Stock Market** ● I can solve problems about the stock market using rational numbers and percentages.
End of Unit Assessment ● Combine mid unit and end of unit (take away 2 coordinate plane questions and add #4 and #6 from mid unit to replace those)		

Unit Title:

Unit 4: Equations and Inequalities (G7 ACC U3)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 10	7.EE.B.3, 7.EE.B.4, 7.EE.B.4.a
Lesson 2	7.EE.B.3	Lesson 11	7.EE.A.27.EE.B.3, 7.EE.B.4, 7.EE.B.4.a
Lesson 3	7.EE.B.3	Lesson 12	6.EE.B.6, 6.EE.B.8, 6.NS.C.7.b
Lesson 4	7.EE.B.3, 7.EE.B.4.a	Lesson 13	6.EE.A.2.b, 6.EE.B.5, 6.EE.B.8
Lesson 5	7.EE.B.3, 7.EE.B.4, 7.EE.B.4.a	Lesson 14	7.EE.B.4.b
Lesson 6	7.EE.B.4.a	Lesson 15	7.EE.B.4
Lesson 7	7.EE.B.4.a	Lesson 16	7.EE.B.4.b
Lesson 8	7.EE.B.4, 7.EE.B.4.a	Lesson 17	7.EE.B.4.b
Lesson 9	7.EE.B.4.a	Lesson 18	7.EE.B.4, 7.G.B.5

Essential Question(s):

- How can visual models like tape diagrams and hanger diagrams be used to solve multi-step equations?
- What does it mean to find a solution to an equation or an inequality?
- How do the solutions to an inequality differ from the solutions to an equation?
- How can I determine the direction of an inequality's solution set?

Enduring Understanding(s):

- Algebraic equations can be solved by performing the same operations on both sides to maintain balance, effectively "undoing" the operations within the expression
- A solution is a value that makes a mathematical statement true; equations typically have one specific solution, while inequalities represent a range of possible values
- While an equation typically has one specific solution, an inequality represents a range of possible values that make the statement true, which is visualized as a shaded region on a number line
- To solve an inequality, find the boundary value by solving the related equation and then test values on either side to see which satisfy the original constraint.

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 5)
CFA 2: Checkpoint B (after lesson 11)
CFA 3: Checkpoint C (after lesson 17)
EoU: Assessment A (after lesson 17)

Pacing for Unit

19 Days
Lesson Modifications:

- Optional lessons: 2 (Lessons 11 and 18)
- Covering Lessons 1-17
- 2 days for review/assessment
- 2 flex days

Family Overview

https://accessim.org/6-8-accelerated/accelerated-7/unit-3?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Aligned Unit Materials, Resources, and Technology

	New Terminology	
Lesson	receptive	productive
Acc7.3.1		equation
Acc7.3.2	unknown amount	
Acc7.3.3	equivalent expressions commutative (property)	expression
Acc7.3.4	solution to an equation	unknown amount relationship
Acc7.3.5		variable
Acc7.3.6	balanced hanger each side (of an equation)	solution to an equation
Acc7.3.7		equivalent expression each side (of an equation)
Acc7.3.8		operation solve
Acc7.3.9	distribute substitute	
Acc7.3.12	inequality maximum minimum	less than greater than
Acc7.3.13	solution to an inequality less than or equal to greater than or equal to open / closed circle	
Acc7.3.14	boundary direction (of an inequality)	less than or equal to greater than or equal to substitute
Acc7.3.15		open / closed circle
Acc7.3.16		solution to an inequality
Acc7.3.17		inequality
Acc7.3.18	perpendicular	
Acc7.3.1		equation

Digital Applets

- 3.2 Every Story Needs a Picture
- 3.3 Drawing Tape Diagrams to Represent Equations
- 3.4 Situations and Diagrams
- 3.5 More Situations and Diagrams

Provide access as needed throughout the unit:

- Blank paper
- Math Community Chart
- Sticky notes
- Tools for creating a visual display

Lesson	Materials to Gather	Materials to
10	• Sticky notes: Activity 3 • Tools for creating a visual display: Activity 3	
11	• Sticky notes: Activity 3 • Tools for creating a visual display: Activity 3	
12	Blank paper: Lesson	Stories about 9 copy for every 2 students): Activi
16	Tools for creating a visual display: Activity 3	
17	Math Community Chart: Activity 2	Giving Advice C copy for every 4 students): Activi

Opportunities for Interdisciplinary Connections:

Finance: Students write equations to model business costs, such as pizza delivery fees

Anticipated misconceptions:

Parentheses in Equations: When solving p(x+q)=r, students often subtract q from r before dividing by p

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

Essential prior concepts to engage with this unit:

- Understanding variables and solving simple equations (Grade 6, Accelerated 6 Unit 4
- Substitution & evaluating expressions (Grade 6)
- The meaning of "solution"
- Adding, subtracting, multiplying, dividing rational numbers (Accelerated 6 Unit 7 or Grade 7)
- Commutative, associative, distributive properties (Grades 3–6)
- Ordering and comparing numbers (K–6)

Relevant Unit(s)/Lesson(s) to Review:

- 6ACC Unit 4 (Equations—lessons 2–4)
- 6ACC Unit 7 (Rational Numbers) Lessons 1–4

Connections to Future Units:

Students will build on this work when they solve equations with a variable on both sides of the equal sign and when they work with systems of equations.

Differentiation through *Universal Design for Learning*

Engagement:
- Provide tools like calculators to facilitate focus on key mathematical ideas in solving inequalities (Lesson 15, Activity 2 Student Task Statement)
  LT6: Solve an inequality of the form px+q>r or px+q<r and interpret the solution

Representation:
- Provide blank templates of tape diagrams labeling the different parts (Lesson 3, Activity 2 Student Task Statement)
  LT1: Create diagrams and equations in the form px+q=r and p(x+q)=r to represent situations

Action & Expression:
- Invite students to plan a strategy, including tools, before starting to solve word problems (Lesson 10, Activity 1 Student Task Statement)
  LT3: Solve equations of the form px+q=r and p(x+q)=r

Supporting Multilingual Learners

Math Language Routines

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MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:
- MLR2: Collect and Display (Lessons 3, 13, 16)
- MLR3: Clarify, Critique, Correct (Lessons 2, 11, 16)
- MLR6: Three Reads (Lessons 1, 11, 17)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret
- Non-proportional situations with constant rates of change (Lessons 1 and 10).
- Solutions to equations (Lessons 4 and 5).
- Equations representing angle measurements (Lesson 18).

Compare
- Stories with corresponding tape diagrams (Lesson 2).
- Tape diagrams with corresponding equations (Lesson 3).
- Hanger diagrams and equations (Lesson 6).
- Solution pathways (especially Lesson 9).
- Descriptions of situations with corresponding inequalities (Lesson 16).

Explain
- Strategies for using hanger diagrams to solve equations (Lesson 7).
- Different strategies for solving equations (Lesson 8) and inequalities (Lesson 14).
- Reasoning about situations, tape diagrams, and equations (Lesson 11).
- How to find unknown angle measurements (Lesson 18).

Sentence Frames and Stems

Section A
- In this situation, I notice _____.
- The equation _____ represents this situation because ...
- To represent this equation, I used _____ (strategy/tool) to show _____ because ...
- The tape diagram represents the equation _____ because ...
- I noticed _____, which means the equation _____ is equivalent to the equation _____.
- The solution to the equation _____ is _____ because ...

Section B
- The equation _____ represents this situation because ...
- To find the unknown weight on the hanger diagram, first I _____, then I ...
- The solution to the equation _____ is _____ because ...
- I began solving the equation by _____ on both sides. Then, I found the solution by _____.
- The solution _____ to the equation _____ makes sense because ...

Section C

- The phrase _____ means _____ and can be represented by the inequality _____.
- The value _____ is a solution to the inequality _____ because ...
- The solution to the inequality _____ can be represented on a number line diagram by ...
- The inequality _____ represents this situation because ...
- The equation _____ represents this situation because ...
- To find the unknown weight on the hanger diagram, first I _____, then I ...
- The solution to the equation _____ is _____ because ...
- I began solving the equation by _____ on both sides. Then, I found the solution by _____.
- The solution _____ to the equation _____ makes sense because ...
- The solution to the inequality _____ can be modeled on a number line diagram by ...
- The solution is _____. This means any number _____ makes the inequality true.
- The value _____ is a solution to the inequality _____ because ...
- The inequality _____ represents this situation because ...

Section D

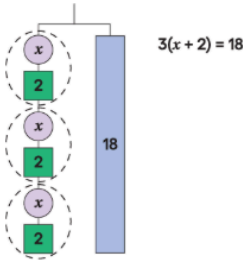
- Angle _____ (name e.g PQR) is an acute/obtuse angle because ...
- Angles _____and _____ (names) are complementary/supplementary angles because ...
- Since angles _____ and _____ (names) are complementary/supplementary, the missing angle measure is _____ because ...
- Lines _____ and _____ (names) cross each other, so angles _____ and _____ (names) are _____ ...
- I used the equation _____ for the relationship between angles_____ and _____ (names) because ...

Unit Outline

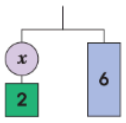
In this unit, students deepen their algebraic reasoning as they write and solve equations of the forms $px+q=r$ and $p(x+q)=r$ and inequalities of the forms $px+q>r$ and $px+q<r$. This builds on grade 6 work with equations of the form $p+x=q$ or $px=q$. Students will build on this work in future units when they solve equations that have a variable on both sides of the equal sign and when they work with systems of equations.

Students begin the unit by making sense of situations that involve both multiplication and addition. They represent such situations with tape diagrams and with equations. They see that different diagrams and equations can represent the same situation, and they use diagrams to find solutions to equations.

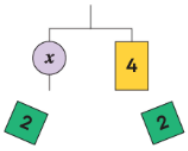
Next, students consider hanger diagrams as another way to represent equations. The diagrams help students understand solving equations in terms of "doing the same thing to each side of the equation." Students examine different pathways for solving the same equation and consider whether one method is more efficient than another.



$3(x+2)=18$



$x+2=6$



$x=4$

Then students apply what they have learned about equations to inequalities. They write inequalities to represent situations and solve inequalities by reasoning about the related equation. The inequality symbols and are introduced.

Lastly, students use what they know about equations to solve problems involving relationships between angles.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Representing Situations of the Form $px+q=r$ and $p(x+q)=r$ (Lessons 1-5)	**Learning Target #1** Create diagrams and equations in the form $px+q=r$ and $p(x+q)=r$ to represent situations. **Learning Target #2** Interpret equations in the form $px+q=r$ and $p(x+q)=r$ that represent relationships in diagrams and situations.	**Lesson 1 Relationships between Quantities** • I can use a tape diagram to find an unknown amount in a situation. • I can think of ways to solve some more complicated word problems. **Lesson 2 Reasoning about Contexts with Tape Diagrams** • I can explain how a tape diagram represents parts of a situation and relationships between them. **Lesson 3 Reasoning about Equations with Tape Diagrams** • I can match equations and tape diagrams that represent the same situation. • If I have an equation, I can draw a tape diagram that shows the same relationship. **Lesson 4 Reasoning about Equations and Tape Diagrams (Part 1)** • I can draw a tape diagram to represent a situation where there is a known amount and several copies of an unknown amount and explain what the parts of the diagram represent.

		• I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true. **Lesson 5 Reasoning about Equations and Tape Diagrams (Part 2)** • I can draw a tape diagram to represent a situation where there is more than one copy of the same sum and explain what the parts of the diagram represent. • I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.
Checkpoint A	***Responding to Student Thinking*** • **Problem 1:** Points to Emphasize: If most students struggle with coordinating the parts of the equation with the diagram, focus on the ways students see each part of the equation when students interpret hanger diagrams in the referenced lessons. ○ Accelerated 7, Unit 3, Lesson 6 Reasoning about Solving Equations (Part 1) ○ Accelerated 7, Unit 3, Lesson 7 Reasoning about Solving Equations (Part 2) • **Problem 2:** More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.	
Section B Solving Equations of the Form $px+q=r$ and $p(x+q)=r$ and Problems That Lead to Those Equations (Lessons 6-11)	**Learning Target #3** Solve equations of the form $px+q=r$ and $p(x+q)=r$, including those that involve fractions, decimals, and negative numbers, and explain the solution method. **Learning Target #4** Solve word problems leading to equations of the form $px+q=r$ or $p(x+q)=r$	**Lesson 6 Reasoning about Solving Equations (Part 1)** • I can explain how a balanced hanger and an equation represent the same situation. • I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram. • I can write an equation that describes the weights on a balanced hanger diagram. **Lesson 7 Reasoning about Solving Equations (Part 2)** • I can explain how a balanced hanger and an equation represent the same situation. • I can explain why some balanced hangers can be represented by two different equations, one with parentheses and one without. • I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram. • I can identify an equation that represents the weights on a balanced hanger diagram. **Lesson 8 Dealing with Negative Numbers** • I can use the idea of doing the same to each side to solve equations that have negative numbers or solutions. **Lesson 9 Different Options for Solving One Equation** • For an equation like , I can solve it in two different ways: by first dividing each side by 3, or by first rewriting using the distributive property. • For equations with more than one way to solve, I can choose the most efficient way depending on the numbers in the equation. **Lesson 10 Using Equations to Solve Problems** • I can solve story problems by drawing and reasoning about a tape diagram or by writing and solving an equation. **Lesson 11 Solving Problems about Percent Increase or Decrease** • I can solve story problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.
Checkpoint B	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of solving equations. If most students struggle with solving equations of the form $px+q=r$, make time to revisit related work in the Sections referred to here. See the Course Guide for ideas to help students re-engage with earlier work. ○ Accelerated 7, Unit 3, Section A Representing Situations of the Form $px+q=r$ and $p(x+q)=r$ ○ Accelerated 7, Unit 3, Section B Solving Equations of the Form $px+q=r$ and $px+q=r$ and Problems That Lead to Those Equations • **Problem 2:** Points to Emphasize: If most students struggle with representing and solving the problem, focus on connecting diagrams and equations to the relationships in problems throughout the next section. For example, invite multiple students to share how the tape diagram matches the stories in the referenced practice problem. Encourage students to use and explain diagrams and equations as needed when representing and solving problems in lesson activities and practice problems throughout the section.	

<table>
<tr><td></td><td colspan="2">○ Grade 7, Unit 6, Lesson 13, Practice Problem 4</td></tr>
<tr><td>Section C
Inequalities
(Lessons 12-17)</td><td>Learning Target #5
Draw and label a graph on a number line that represents all the solutions to an inequality.

Learning Target #6
Solve an inequality of the form px+q>r or px+q<r and interpret the solution.

Learning Target #7
Write an inequality of the form px+q>r or px+q<r to represent a situation with a constraint.</td><td>Lesson 12 Writing and Graphing Inequalities
● I can graph inequalities on a number line.
● I can write an inequality to represent a situation.
Lesson 13 Solutions of Inequalities
● I can explain what it means for a number to be a solution to an inequality.
● I can explain what the symbols ≥ and ≤ mean.
● I can graph the solutions to an inequality on a number line.
Lesson 14 Finding Solutions to Inequalities in Context
● I can describe the solutions to an inequality by solving a related equation and then reasoning about values that make the inequality true.
● I can write an inequality to represent a situation.
Lesson 15 Efficiently Solving Inequalities
● I can graph the solutions to an inequality on a number line.
● I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality.
Lesson 16 Interpreting Inequalities
● I can match an inequality to a situation it represents, solve it, and then explain what the
● solution means in the situation.
● If I have a situation and an inequality that represents it, I can explain what the parts of the inequality mean in the situation.
Lesson 17 Modeling with Inequalities
● I can use what I know about inequalities to solve real-world problems.</td></tr>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
● Problem 1: Press Pause: By this point in the unit, there should be some student mastery of representing inequalities on a number line. If most students struggle, make time to revisit related work in the referenced lessons. See the Course Guide for ideas to help students re-engage with earlier work.
○ Accelerated 7, Unit 3, Lesson 12 Writing and Graphing Inequalities
○ Accelerated 7, Unit 3, Lesson 13 Solutions of Inequalities
● Problem 2: Press Pause: By this point in the unit, there should be some student mastery of writing and solving inequalities. If most students struggle, make time to revisit related work in the referenced sections. See the Course Guide for ideas to help students re-engage with earlier work.
○ Accelerated 7, Unit 3, Section B Solving Equations of the Form px+q=r and p(x + q)= r and Problems That Lead to Those Equations.
○ Accelerated 7, Unit 3, Section C Inequalities</td></tr>
<tr><td>Section D
Let's Put it To Work
(Lesson 18)</td><td>No new learning targets</td><td>Lesson 18 Using Equations to Solve for Unknown Angles
● I can write an equation to represent a relationship between angle measures and solve the equation to find unknown angle measures.</td></tr>
<tr><td colspan="3">End of Unit Assessment</td></tr>
</table>

Unit Title:

Unit 5: Expressions and More Equations (G7 ACC U4)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	7.EE.A.1, 7.NS.A.1, 7.NS.A.1.c	Lesson 7	8.EE.C, 8.EE.C.7
Lesson 2	7.EE.A.1	Lesson 8	8.EE.C, 8.EE.C.7
Lesson 3	7.EE.A.1	Lesson 9	8.EE.C.7, 8.EE.C.7.b
Lesson 4	7.EE.A.1	Lesson 10	8.EE.C.7.a
Lesson 5	7.EE.a, 8.EE.C	Lesson 11	8.EE.C.7.a
Lesson 6	8.EE.C, 8.EE.C.7	Lesson 12	–

Essential Question(s):

- How can we prove that two algebraic expressions are equivalent?
- What algebraic "moves" keep an equation balanced when solving?
- How can the structure of an equation tell us how many solutions it has?

Enduring Understanding(s):

- Expressions are equivalent if they have the same value for any variable substitution; this equivalence is maintained using the distributive, commutative, and associative properties to combine like terms
- Solving equations with variables on both sides involves using "balanced moves" to gather all variable terms on one side and constant terms on the other while maintaining equality
- An equation can have one solution, no solution (if it implies two different values are equal), or infinitely many solutions (if both sides are identical)

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 4)
CFA 2: Checkpoint B (after lesson 9)
CFA 3: Checkpoint C (after lesson 11)
EoU: Assessment A (after lesson 11)

Pacing for Unit

13 Days
Lesson Modifications:

- Optional lessons: 1 (Lesson 12)
- Covering Lessons 1-11
- 2 days for review/assessment

Family Overview

https://accessim.org/6-8-accelerated/accelerated-7/unit-4?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
Acc7.4.1	term	
Acc7.4.2	factor (an expression) expand (an expression)	
Acc7.4.3	combine like terms	term commutative (property)
Acc7.4.4		distribute
Acc7.4.8	distributive property	
Acc7.4.9		like terms common denominator
Acc7.4.10	no solution (only) one solution	
Acc7.4.11	constant term coefficient infinitely many solutions	

Aligned Unit Materials, Resources, and Technology

Provide access as needed throughout the unit:

- Index cards
- Math Community Chart

Lesson	Materials to Gather	Materials to Copy
4	Index cards: Warm-up	
6	Math Community Chart: Activity 1	Matching Equation Moves Cards (1 copy for every 2 students): Activity 1
8		Trading Moves Cards (1 copy for every 2 students): Activity 2
11		Thinking About Solutions Some More Cards (1 copy for every 3 students): Activity 1

Opportunities for Interdisciplinary Connections:

Earth Science: Algebraic operations are performed

Anticipated misconceptions:

Variable Consistency: Students may forget that a

within the context of a gold mine	variable (like x) must represent the same value throughout a single problem or diagram See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:	**Connections to Future Units:**
Essential prior concepts to engage with this unit: • Writing and evaluating expressions; equivalent equations (Grade 6, Accelerated 6 Unit 4) • Combining like terms (Grade 6–7) • Distributive property (Grade 3, formalized in Grade 6–7) • Solving px+q=r (Grade 7 Unit 3 or Accelerated 6 Unit 4) • Understanding solution types (Unit 3) • Fluency with signed numbers (Accelerated 6 Unit 7) **Relevant Unit(s)/Lesson(s) to Review:** • 6ACC Unit 4, Lessons 7–9 • 6ACC Unit 7, Lesson 6–10	This work builds toward solving systems of linear equations in future units.

Differentiation through *Universal Design for Learning*

Engagement:
- Differentiate the degree of difficulty by starting with more accessible expressions like 6x−(2x+8) (Lesson 4, Activity 2 Student Task Statement)
  LT1: Apply properties of operations to write an expression with fewer terms

Representation:
- Use color coding and annotations to highlight connections between a hanger diagram and its corresponding equation (Lesson 6, Activity 1 Launch)
  LT3: Write equivalent equations and describe the moves used

Action & Expression:
- Provide students with a blank two-column table to keep track of moves and variables in number puzzles (Lesson 8, Activity 3 Launch)
  LT4: Write equivalent equations to solve linear equations in one variable

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Displa*y - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:
- MLR1: Stronger and Clearer Each Time (Lessons 1, 11)
- MLR7: Compare and Connect (Lessons 4, 6, 7, 8, 12)
- MLR8: Discussion Supports (Lessons 1, 2, 3, 4, 6, 8, 9, 10, 11)

Progression of Interdisciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as critiquing, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique
- Reasoning of peers about expressions and corresponding diagrams (Lesson 1).
- Reasoning about equivalent expressions (Lesson 4).
- Reasoning about maintaining balance in equations (Lesson 6).
- Solutions of linear equations (Lessons 7 and 8).

Justify
- Reasoning about the distributive property (Lesson 2).
- Strategies for writing equivalent equations (Lesson 8).

- Predictions about maintaining balance (Lesson 5).
- Predictions about solutions of linear equations (Lesson 9).
- Whether different sequences of calculations give the same result (Lesson 12).

Generalize

- About when expressions are equivalent (Lesson 3).
- About the structures of equations that have one, infinite, and no solutions (Lessons 10 and 11).

Sentence Frames and Stems

Section A

- I used the _____ property to simplify the expression from _____ to _____.
- I combined like terms … to simplify the expression. My new expression is _____.
- The distributive property allowed me to expand/factor the expression _____ to _____.
- The expression _____ is equivalent to the expression _____ because …

Section B

- The moves … show that the equation _____ is equivalent to the equation _____.
- To solve the equation _____, the first move was to _____ on both sides, then …
- To find the unknown weight on the hanger diagram, first I _____, then I …
- I know that equation _____ will have a positive/negative/zero solution because …

Section C

- The equation _____ has _____ solution(s). I know this because …
- The equation _____ represents …
- The solution _____ to the equation _____ makes sense because …

Section D

- I used the expression_____ to represent the situation _____ because …
- The expression _____ and the expression _____ are equivalent.

Unit Outline

In this unit, students work with writing equivalent expressions and use reasoning to solve equations, including equations that have a variable on both sides of the equal sign. This builds on students' previous work solving equations of the form px+q=r or p(x+q)=r. Students will build on this work in future units when they solve systems of linear equations.

First, students work with equivalent linear expressions that are more complex due to having more terms, more parentheses, and negative rational numbers. Students use properties of operations to justify why the expressions are equivalent.

Next, the unit focuses on moves that can be done to write equivalent equations. At first, students use hanger diagrams as an intuitive representation of equality and represent their reasoning by labeling arrows that connect equivalent representations. With the reintroduction of negative values, students move away from hanger diagrams to algebraic equations and writing equivalent equations with the intention of solving for a variable.

$$9-2b+6=-3(b+5)+4b$$

Use the distributive property	$9-2b+6=-3b-15+4b$
Combine like terms	$15-2b=b-15$
Add $2b$ to each side	$15=3b-15$
Add 15 to each side	$30=3b$
Divide each side by 3	$10=b$

Lastly, students examine the conditions under which equations could have 0, 1, or infinite solutions as a transition to thinking about similar situations involving systems of equations.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Writing Equivalent Expressions (Lessons 1-4)	**Learning Target #1** Apply properties of operations to write an expression with fewer terms that is equivalent to a given expression **Learning Target #2** Apply the distributive property to factor or expand an expression	**Lesson 1 Subtraction in Equivalent Expressions** • I can organize my work when I use the distributive property. • I can rewrite subtraction as adding the opposite and then rearrange terms in an expression. **Lesson 2 Expanding and Factoring** • I can organize my work when I use the distributive property. • I can use the distributive property to rewrite expressions with positive and negative numbers. • I understand that "factoring" and "expanding" are words used to describe using the distributive property to write equivalent expressions. **Lesson 3 Combining Like Terms (Part 1)** • I can figure out whether two expressions are equivalent to each other. • When possible, I can write an equivalent expression that has fewer terms.

		Lesson 4 Combining Like Terms (Part 2) ● I am aware of some common errors when writing equivalent expressions, and I can avoid them. ● When possible, I can write an equivalent expression that has fewer terms.
Checkpoint A	***Responding to Student Thinking*** ● **Problem 1:** Points to Emphasize: If students struggle with writing an equivalent expression with fewer terms, revisit this concept when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share their thinking about these practice problems: ○ Accelerated 7, Unit 4, Lesson 5, Practice Problem 4 ○ Accelerated 7, Unit 4, Lesson 6, Practice Problem 5 ● **Problem 2:** Points to Emphasize: If students struggle with using the distributive property to factor or expand an expression, revisit this concept when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share their thinking about these practice problems: ○ Accelerated 7, Unit 4, Lesson 6, Practice Problem 6 ○ Accelerated 7, Unit 4, Lesson 7, Practice Problem 5	
Section B Equivalent Expressions (Lessons 5-9)	**Learning Target #3** Write equivalent equations and describe the moves that are used. **Learning Target #4** Write equivalent equations to solve equations in one variable.	**Lesson 5 Keeping the Equation Balanced** ● I can add or remove blocks from a hanger and keep the hanger balanced. ● I can represent balanced hangers with equations. **Lesson 6 Balanced Moves** ● I can add, subtract, multiply, or divide each side of an equation by the same expression to get a new equation with the same solution. **Lesson 7 More Balanced Moves** ● I can make sense of multiple ways to solve an equation. **Lesson 8 Solving any Linear Equation** ● I can solve an equation where the variable appears on both sides. **Lesson 9 Strategic Solving** ● I can solve linear equations in one variable.
Checkpoint B	***Responding to Student Thinking*** ● **Problem 1:** More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. ● **Problem 2:** More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.	
Section C Linear Equations in One Variable (Lessons 10-11)	**Learning Target #5** Describe features of linear equations that have one solution, no solution, or many solutions. **Learning Target #6** Interpret the solution of an equation in one variable in context.	**Lesson 10 All, Some, or No Solutions** ● I can determine whether an equation has no solutions, one solution, or infinitely many solutions. **Lesson 11 How Many Solutions?** ● I can solve equations with different numbers of solutions.
Checkpoint C	***Responding to Student Thinking*** ● **Problem 1:** Points to Emphasize: If students struggle to connect an equation to the number of solutions, revisit the idea when a similar situation arises for systems of equations. For example, the inclusion of graphs to see why there are no solutions can help make the connection between coefficients and slope. ○ Accelerated 7, Unit 5, Lesson 15 Solving Systems of Equations ● **Problem 2:** Points to Emphasize: If students struggle to solve the equation, make time for students to practice solving linear equations, especially those involving the distributive property. For example, encourage students to fully solve this practice problem: ○ Accelerated 7, Unit 5, Lesson 1, Practice Problem 3	
Section D Let's Put it To Work (Lesson 12)	**No new learning targets**	**Lesson 12 Applications of Expressions** ● I can write algebraic expressions to understand and justify a choice between two options.
End of Unit Assessment		

Unit Title:

Unit 6: Rigid Transformations & Congruence (G7 ACC U1)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 10	8.G.A.1, 8.G.A.2
Lesson 2	8.G.A.1	Lesson 11	8.G.A.1.a, 8.G.A.2
Lesson 3	8.G.A.1	Lesson 12	7.G.B.5, 8.G.A.1, 8.G.A.5
Lesson 4	8.G.A.3	Lesson 13	8.G.A.2, 8.G.A.5
Lesson 5	8.G.A.1, 8.G.A.3	Lesson 14	8.G.A.5
Lesson 6	8.G.A.1.a, 8.G.A.1.b	Lesson 15	7.G.A.2
Lesson 7	8.G.A.1.a, 8.G.A.1.b	Lesson 16	7.G.A.2
Lesson 8	8.G.A.1.a, 8.G.A.1.b, 8.G.A.1.c	Lesson 17	7.G.A.2
Lesson 9	8.G.A.1.a, 8.G.A.1.b	Lesson 18	8.G.A

Essential Question(s):

- How do translations, rotations, and reflections affect the properties of a figure?
- What does it mean for two geometric figures to be "congruent"?
- What universal relationship exists among the interior angles of any triangle?

Enduring Understanding(s):

- Rigid transformations change a figure's position in the plane but preserve all side lengths and angle measures
- Two figures are congruent if there is a sequence of rigid transformations that takes one figure exactly onto the other
- The sum of the interior angle measures of any triangle is always 180 degrees, a fact that can be demonstrated through rigid motions or parallel line relationships

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 5)
CFA 2: Checkpoint B (after lesson 9)
MoU: Assessment A (after lesson 9)
Checkpoint C is an opportunity for feedback
CFA 3: Checkpoint D (after lesson 14)
CFA 4: Checkpoint E (after lesson 17)
EoU: Assessment A (after lesson 17)

Pacing for Unit

17 Days
Lesson Modifications:

- Covering Lessons 1-17
- Omit Lesson 1, 6, 9, 10
- Omit Lesson 10 (one of two congruence lessons)
- Add complementary and supplementary angles to the end of unit (Lesson 12 and 13)
- Could omit Lesson 15 (building polygons)
- Combine 16 and 17 in one day (drawing triangles)
- 4 days for review/assess

Family Overview

https://accessim.org/6-8-accelerated/accelerated-7/unit-1?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
Acc7.1.1	vertex plane measure direction figure	slide turn
Acc7.1.2	clockwise counterclockwise reflection rotation translation original	opposite
Acc7.1.3	image	vertex

Aligned Unit Materials, Resources, and Technology

Digital Applets

- 1.3 Image Information, A to B to C
- 1.6 Sides and Angles, Which One?
- 1.7 A Pattern of Four Triangles
- 1.15 Where is Lin? Building Diego's and Jada's Shapes, How Long is the Third Side?
- 1.16 How Many Can You Draw?, Revisiting How Many Can You Draw
- 1.17 Revisiting How Many Can You Draw, Three Angles

Provide access as needed throughout the unit:

- Chart paper
- Compasses
- Geometry toolkits

	angle of rotation center (of rotation) line of reflection transformations sequence of transformations distance	clockwise counterclockwise reflect rotate translate
Acc7.1.4	coordinate plane point segment coordinates x-axis y-axis	
Acc7.1.5	polygon	angle of rotation center (of rotation) line of reflection
Acc7.1.6	rigid transformation corresponding measurements preserve	reflection rotation translation measure point
Acc7.1.7	midpoint	segment
Acc7.1.8	vertical angles parallel intersect	distance
Acc7.1.9		image rigid transformation midpoint parallel
Acc7.1.10	congruent perimeter area	
Acc7.1.11		right angle x-axis y-axis area corresponding
Acc7.1.12	alternate interior angles transversal supplementary complementary	vertical angles congruent
Acc7.1.13	straight angle	supplementary
Acc7.1.14		alternate interior angles transversal straight angle
Acc7.1.15	identical copy condition compass different triangle	
Acc7.1.16	unique triangle	condition different triangle
Acc7.1.17		protractor compass
Acc7.1.18	tessellation symmetry	

- Math Community Chart
- Metal paper fasteners (brass brads)
- Rulers
- Scissors
- Sticky notes
- Tracing paper

Lesson	Materials to Gather	Materials to Copy
1	• Geometry toolkits: Lesson, Warm-up • Chart paper: Warm-up • Sticky notes: Warm-up	• 6-12 Blank Math Community Chart (1 copy for every 30 students): Warm-up • Triangle Square Dance Handout (1 copy for every 2 students): Activity 1
2	• Geometry toolkits: Warm-up, Activity 1 • Math Community Chart: Warm-up • Sticky notes: Warm-up	Move Cards (1 copy for every 3 students): Activity 2
3	• Geometry toolkits: Activity 1, Activity 2	
4	• Geometry toolkits: Activity 2	
5	• Geometry toolkits: Warm-up • Math Community Chart: Warm-up	Transformation Information Cards (1 copy for every 2 students): Activity 1
6	Geometry toolkits: Activity 2	
7	• Math Community Chart: Lesson, Warm-up • Sticky notes: Warm-up • Geometry toolkits: Activity 1, Activity 2	
8	• Tracing paper: Warm-up, Activity 1 • Geometry toolkits: Activity 2	
9	Geometry toolkits: Warm-up, Activity 1, Activity 2, Activity 3	
10	• Geometry toolkits: Warm-up, Activity 1, Activity 2 • Math Community Chart: Warm-up, Activity 1	
11	• Chart paper: Warm-up • Geometry toolkits: Warm-up, Activity 1, Activity 2 • Math Community Chart: Warm-up • Rulers: Activity 2	

	12	• Geometry toolkits: Warm-up, Activity 1, Activity 2 • Math Community Chart: Activity 4	Angle Finding Cards (1 copy for every 2 students): Activity 4
	13	• Sticky notes: Warm-up • Geometry toolkits: Activity 1, Activity 2 • Scissors: Activity 2	• Find All Three Cards (1 copy for every 2 students): Activity 4 • Tear it Up Cards (1 copy for every 4 students): Activity 2
	14	Geometry toolkits: Activity 1	
	15	• Geometry toolkits: Warm-up, Activity 1, Activity 2 • Metal paper fasteners: Activity 1, Activity 2 • Compasses: Activity 2	What Can You Build? Cutouts (1 copy for every 2 students): Activity 1
	16	Geometry toolkits: Activity 1, Activity 2	
	17	• Compasses: Warm-up, Activity 1, Activity 2 • Geometry toolkits: Activity 1, Activity 2	Revisiting How Many Can You Draw? Handout (1 copy for each student): Activity 1
	18	Geometry toolkits: Warm-up, Activity 1, Activity 2	Deducing Angle Measures Handout (1 copy for every 2 students): Warm-up

Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions:**
Computer Science: The unit uses video game movement to introduce translations, rotations, and reflections	**Clockwise/Counterclockwise:** Students often confuse the direction of rotation, especially when it is not explicitly demonstrated See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:	**Connections to Future Units:**
Essential prior concepts to engage with this unit: • Understanding angles and angle measure (Grade 4) • Angle relationships (Grade 4–5) • Coordinate planes (Grade 5) • Measuring distance (Grades K–5) • Polygons and their properties (Grade 5) • Area and perimeter (Grades 3–5) • Visualizing and describing movements (K–2) **Relevant Unit(s)/Lesson(s) to Review:** • Brief review of angle measurement (protractor use), coordinate plotting in all quadrants, and polygon naming. Focus on fluency, not conceptual depth, so students can participate in Unit 1 activities without tool/notation barriers.	Builds toward exploring dilations and similar figures in the plane. Lays foundation for high school geometry congruence proofs.

Differentiation through *Universal Design for Learning*

Engagement:
- Use visible timers to help learners anticipate transitions between sharing strategies (Lesson 4, Activity 1 Launch)
  LT1: Determine coordinates that represent the image after a transformation

Representation:
- Begin with a physical demonstration using tracing paper to perform transformations (Lesson 3, Activity 1 Launch)

LT2: Draw and label the image of figures from transformations

Action & Expression:

- Invite students to talk about their ideas with a partner before writing them down (Lesson 2, Activity 1 Launch)
  LT2: Draw and label the image of figures from transformations

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Displa*y - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 7, 8, 9, 11, 17)
- MLR2: Collect and Display (Lessons 1, 2, 6, 10, 12, 16)
- MLR8: Discussion Supports (Lessons 2, 3, 4, 6, 7, 12, 14, 15)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, generalizing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Movements of figures (Lessons 1 and 2).
- Observations about transforming parallel lines (Lesson 8).
- Transformations using corresponding points, line segments, and angles (Lesson 9).
- Observations about angle measurements (Lesson 14).
- Transformations found in tessellations and in designs with rotational symmetry (Lesson 18).

Generalize

- About categories for movement (Lesson 2).
- About rotating line segments 180° (Lesson 7).
- About the relationship between vertical angles (Lesson 8).
- About transformations and congruence (Lesson 11).
- About corresponding segments and length (Lesson 11).
- About alternate interior angles (Lesson 12).
- About the sum of angles in a triangle (Lesson 14).
- About categories for unique triangles (Lesson 16).

Justify

- Whether or not rigid transformations could produce an image (Lesson 6).
- Whether or not shapes are congruent (Lesson 10).
- Whether or not polygons are congruent (Lesson 11).
- Whether or not triangles can be created from given angle measurements (Lesson 13).
- Whether or not measurements determine unique triangles (Lesson 17).

Sentence Frames and Stems

Section A

- The coordinates of the image are ... after a _____ (transformation).
- When a figure is transformed using a _____ (transformation), the coordinates change by ...
- A _____ (transformation) of a figure on a grid looks like ...
- To draw an image of figure _____ using a _____ (transformation), first I _____, then I ...

Section B

- Between the original figure _____ and its image, side _____ corresponds to side _____ ...
- The image of line _____ is _____ to the original because ...
- I know the image was created using a _____ (transformation) because ...

Section C

- Figure _____ is/is not congruent to figure _____ because ...
- I can prove figure _____ is congruent to figure _____ by using the following transformations ...

Section D

- Given the parallel lines _____ and transversal _____, angles _____ and _____ are congruent because ...
- If I know the measure of angle _____ is _____ degrees, then angle _____ must be _____ degrees because ...
- The sum of the measures of angles _____ and _____ is _____ degrees because ...
- To find the angle measures in the triangle, first I _____, then I ...

Section E

- Side lengths _____, _____ and _____ will/will not form a triangle because ...
- Two side lengths of the triangle are _____ and _____. The length of the third side of the triangle will be longer than _____ but shorter than _____ because ...
- Triangles _____ and _____ (names) are/ are not identical because ...
- I think that angles _____ and _____ (names) with side length _____ will/will not form a unique triangle because ...

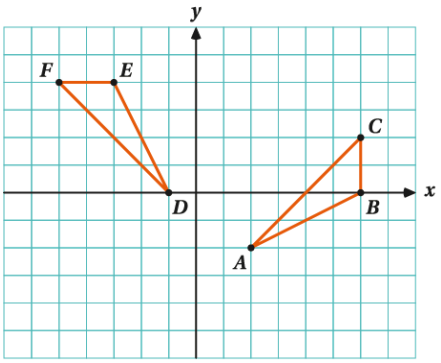
Section F

- To create a tessellation, first I _____, then I ...
- _____ are shapes that work well in tessellations because ...
- To create a figure with rotational symmetry, I have to think about ...

Unit Outline

In this unit, students explore translations, rotations, and reflections of plane figures in order to understand the structure of rigid transformations. They use the properties of rigid transformations to formally define what it means for shapes to be congruent.

In earlier grades, students studied geometric measurement to find angle measures and side lengths of two-dimensional figures as well as applied area and perimeter formulas for polygons including rectangles, parallelograms, and triangles.



In this unit, students build on this work as they identify corresponding congruent angles and side lengths of figures and their images under rigid transformations. In an upcoming unit, students will explore dilations and similar figures in the plane.

In the first section, students begin with an informal exploration of transformations in the plane, then increase their precision of language to describe translations, rotations, and reflections with formal descriptions, including coordinates.

Then students identify corresponding parts of figures and conclude that angles and distances are preserved under rigid transformations. Students use this property to reason about plane figures, including parallel lines cut by a transversal.

Students then learn the formal definition of "congruent" and use this definition to show that corresponding parts of congruent figures are also congruent. Students apply their understanding of congruence and rigid motions to justify that the sum of the interior angles in a triangle must be 180°.

Students investigate whether sets of angle and side length measurements determine unique triangles or multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms “complementary,” “supplementary,” “vertical angles,” and “unique.”

Note: It is not expected that students memorize which conditions result in a unique triangle, an impossible-to-create triangle, or multiple possible triangles. Understanding that, for example, side-side-side (SSS) information results in zero or exactly one triangle will be explored in high school geometry. At this level, students should attempt to draw triangles with the given information and notice that there is only one way to do it (or that it is impossible to do). In this unit, students reason about congruence and justify properties of figures using rigid transformations, but they are not required to create a formal proof. They will prove these and other geometric properties more formally in later courses.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Rigid Transformations (Lessons 1-5)	**Learning Target #1** Determine coordinates that represent the image of a polygon or line segment in the coordinate plane after a transformation. **Learning Target #2** Draw and label the image of figures that result from translations, rotations, and reflections on a square or isometric grid. **Learning Target #3**	**Lesson 1 Moving in the Plane** • I can describe how a figure moves and turns to get from one position to another. **Lesson 2 Naming the Moves** • I can identify corresponding points before and after a transformation. • I know the difference between translations, rotations, and reflections. **Lesson 3 Making the Moves** • I can use grids to carry out transformations of figures. • I can use the terms translation, rotation, and reflection to precisel describe transformations. **Lesson 4 Coordinate Moves** • I can apply transformations to points on a grid if I know their coordinates.

	Explain the sequence of transformations that takes one figure to its image.	**Lesson 5 Describing Transformations** ● I can apply transformations to a polygon on a grid if I know the coordinates of its vertices.
Checkpoint A	***Responding to Student Thinking*** ● **Problem 1:** Points to Emphasize: If students struggle with identifying coordinates of a point or line segment after a translation or reflection, spend time addressing this in a future lesson. For example, the Activity Synthesis of the activity referred to here, draw a set of axes on a grid and ask students t identify the coordinates of a few points on the figure and its image. ○ Accelerated 7, Unit 1, Lesson 6,Activity 1 Sides and Angles ● **Problem 2:** Points to Emphasize: If students struggle with describing a translation, rotation, or reflection, spend time in a future lesson addressing the description of a sequence of transformation For example, in the activity referred to here, ask students to describe the sequence of transformatio needed to get from the first figure to the second. ○ Accelerated 7, Unit 1, Lesson 7, Warm-up Notice and Wonder:Building a Quadrilateral	
Section B Properties of Rigid Transformations (Lessons 6-9)	**Learning Target #4** Draw and label rigid transformations of lines and parallel lines and explain the relationship between the original and its image under the transformation. **Learning Target #5** Identify a rigid transformation using a drawing of a figure and its image. **Learning Target #6** Identify side lengths and angles that have equivalent measurements in composite shapes and explain why they are equivalent.	**Lesson 6 No Bending or Stretching** ● I can describe the effects of a rigid transformation on the lengths and angles in a polygon. **Lesson 7 Rotation Patterns** ● I can describe how to move one part of a figure to another using a rigid transformation. **Lesson 8 Moves in Parallel** ● I can describe the effects of a rigid transformation on a pair of parallel lines. ● If I have a pair of vertical angles and know the angle measure of o of them, I can find the angle measure of the other. **Lesson 9 Composing Figures** ● I can find missing side lengths or angle measures using propertie rigid transformations.
Checkpoint B	***Responding to Student Thinking*** ● **Problem 1**: Points to Emphasize: If students struggle with constructing the image of a segment after a rotation, spend time in a future lesson addressing strategies for constructing transformations. For example, in the activity referred to here, select students to demonstrate or describe their strategy for rotating the congruent figures. ○ Accelerated 7, Unit 1, Lesson 11, Activity 1 Congruent Pairs ● **Problem 2:** More Chances: Students will have more opportunities to understand the mathematical idea addressed here. There is no need to slow down or add additional work to the next lessons.	
Section C Congruence (Lessons 10-11)	**Learning Target #7** Compare and contrast side lengths, angle measures, and other features of shapes using rigid transformations to explain why a shape is or is not congruent to another. **Learning Target #8** Justify that two polygons on a grid are congruent using the definition of congruence in terms of rigid transformations.	**Lesson 10 What is the Same?** ● I can decide whether or not two figures are congruent using rigid transformations. **Lesson 11 Congruence** ● I can decide using rigid transformations whether or not two figure are congruent. ● I can use distances between points to decide if two figures are congruent.
Checkpoint C	***Responding to Student Thinking*** ● **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of describing ri transformations between congruent figures and justifying why two figures are not congruent. If most students struggle with these concepts, make time to examine related work in the section referred to he The Course Guide provides additional ideas for revisiting earlier work. ○ Accelerated 7, Unit 1, Section C Congruence	
Mid-Unit Assessment		
Section D	**Learning Target #9**	**Lesson 12 Alternate Interior Angles**

Angles in a Triangle (Lessons 12-16)	Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems. **Learning Target #10** Generalize that the sum of angles in a triangle is 180 degrees using rigid transformations or the congruence of alternate interior angles of parallel lines cut by a transversal.	• I can find unknown angle measures by reasoning about complementary or supplementary angles. • I can recognize when adjacent angles are complementary or supplementary. • If I have two parallel lines cut by a transversal, I can identify altern interior angles and use that to find missing angle measurements. **Lesson 13 Adding the Angles in a Triangle** • I can determine whether three angles could make a triangle using their sum. **Lesson 14 Parallel Lines and the Angles in a Triangle** • I can explain using pictures why the sum of the angles in any trian is 180 degrees.
Checkpoint D	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of solving problems using corresponding angles and properties of parallel lines. If students struggle with these concepts, make time to examine related work in the section referred to here. The Course Guide provide additional ideas for revisiting earlier work. ○ Accelerated 7, Unit 1, Section D Angles in a Triangle • **Problem 2:** More Chances: Students will have more opportunities to understand the mathematical idea addressed here. There is no need to slow down or add additional work to the next lessons.	
Section E Drawing Polygons with Given Conditions (Lessons 17-18)	**Learning Target #11** Draw triangles with two given angle measures and one side length, one given angle measure and two side lengths, or three side lengths. **Learning Target #12** Justify whether 3 measures of angles or sides determine a unique triangle or more than one triangle, or if no triangle is possible.	**Lesson 15 Building Polygons** • I can show that the 3 side lengths that form a triangle cannot be rearranged to form a different triangle. • I can show that the 4 side lengths that form a quadrilateral can be rearranged to form different quadrilaterals. • I can show whether or not 3 side lengths will make a triangle. **Lesson 16 Drawing Triangles (Part 1)** • Given two angle measures and one side length, I can draw differe triangles with these measurements or show that these measurements determine one unique triangle or no triangle. **Lesson 17 Drawing Triangles (Part 2)** • Given two side lengths and one angle measure, I can draw differe triangles with these measurements or show that these measurements determine one unique triangle or no triangle.
Checkpoint E	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of creating triangles with 3 given measures. If most students struggle, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work • **Problem 2:** More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this tim	
Section F Let's Put it To Work (Lesson 19)	**No new learning targets**	**Lesson 18 Rotate and Tessellate** • I can repeatedly use rigid transformations to make interesting repeating patterns of figures. • I can use properties of angle sums to reason about how figures w fit together.
End of Unit Assessment		

Unit Title:

Unit 7: Scale Drawings, Similarity, & Slope (G7 ACC U2)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	7.G.A.1	Lesson 11	8.G.a, 8.G.A.3
Lesson 2	7.G.A.1	Lesson 12	8.G.A.4
Lesson 3	7.G.A.1	Lesson 13	8.G.a, 8.G.A.5
Lesson 4	7.G.A.1	Lesson 14	8.G.a, 8.G.A.4
Lesson 5	7.G.A.1	Lesson 15	8.EE.B.6
Lesson 6	7.G.A.1	Lesson 16	8.EE.B.6
Lesson 7	7.G.A.1	Lesson 17	8.EE.B.6, 8.G.a, 8.G.A.3
Lesson 8	–	Lesson 18	7.G.A.1
Lesson 9	8.G.A	Lesson 19	8.G.A.5
Lesson 10	8.G.a, 8.G.A.3		

Essential Question(s):

- How is a dilation different from a rigid transformation?
- What defines similarity between two geometric figures?
- How does triangle similarity explain the concept of slope?

Enduring Understanding(s):

- Unlike rigid transformations, a dilation changes the size of a figure based on a scale factor and a center of dilation, while still keeping the shape (angles) the same
- Two figures are similar if one can be transformed into the other using a sequence of rigid transformations and dilations
- The slope of a line is constant because any two "slope triangles" drawn on the same line are similar, meaning the ratio of vertical change to horizontal change is always the same

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 2)
Checkpoint B (after lesson 5) is an opportunity for feedback
CFA 2: Checkpoint C (after Lesson 10)
CFA 3: Checkpoint D (after lesson 17)
EoU: Assessment A (after lesson 17)

Pacing for Unit

10 Days
Lesson Modifications:

- Lessons to cover: 2,4,5,10, 12,15, 16, 17
- Optional lessons: 1 (Lesson 18)
- 2 days for review/assessment

Family Overview

https://accessim.org/6-8-accelerated/accelerated-7/unit-2?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
Acc7.2.1	scaled copy	
Acc7.2.2	scale factor	
Acc7.2.3	reciprocal measurement	scale factor original
Acc7.2.4	scale drawing scale two-dimensional three-dimensional represent actual	scaled copy
Acc7.2.5	floor plan	scale

Aligned Unit Materials, Resources, and Technology

Digital Applets

- 2.1 Printing Portraits, Scaling F, Pairs of Scaled Polygons
- 2.9 Quadrilateral on a Circular Grid, Getting Perspective
- 2.11 Notice and Wonder, Many Dilations of a Triangle
- 2.12 Similarity Transformations (Part 1)

Provide access as needed throughout the unit:

- Blank paper
- Dried linguine pasta (We specified linguine since it is flatter and less likely to roll around than spaghetti.)
- Geometry toolkits
- Graph paper
- Long straightedge
- Math Community Chart

Acc7.2.6	appropriate dimension	
Acc7.2.7	scale without units _ to _ equivalent scales	scale drawing
Acc7.2.8	scaling	
Acc7.2.9	dilation center of dilation dilate	
Acc7.2.10		center of a dilation
Acc7.2.12	similar	dilation dilate
Acc7.2.14	quotient	
Acc7.2.15		slope slope triangle
Acc7.2.16	similarity x-coordinate y-coordinate equation of a line	quotient
Acc7.2.19	estimate approximate / approximately	

- Measuring tapes
- Measuring tools
- Metric and customary unit conversion charts
- Protractors
- Rulers
- Scissors
- Straightedges
- Tape
- Tracing paper
- Yardsticks

Lesson	Materials to Copy	Materials to Gather
1		• Pairs of Scaled Polygons Cards (1 copy for every 2 students): Activity 2
3	• Protractors: Activity 1 • Geometry toolkits: Activity 3	• Scaled Copies Cards (1 copy for every 3 students): Activity 2 • Scaling A Puzzle Cutouts (1 copy for every 3 students): Activity 3
4	• Geometry toolkits: Activity 1, Activity 2	• Sizing Up a Basketball Court Handout (1 copy for each student): Activity 1
5	• Geometry toolkits: Activity 2	
6	• Geometry toolkits: Activity 2	• Same Plot, Different Drawings Cards (1 copy for every 24 students): Activity 1
7	• Geometry toolkits: Activity 1 • Metric and customary unit conversion charts: Activity 2	• Units of Length Reference Sheet (1 copy for every 2 students): Activity 2
8	• Math Community Chart: Warm-up • Blank paper: Activity 1 • Long straightedge: Activity 1 • Scissors: Activity 1 • Rulers: Activity 2	
9	• Straightedges: Activity 1 • Geometry toolkits: Activity 2, Activity 3	
10	• Geometry toolkits: Warm-up, Activity 1	• Matching Dilations on a Coordinate Plane Cards (1 copy for every 2 students): Activity 2
11	• Math Community Chart: Warm-up • Geometry toolkits: Activity 1	• Dilation Cards (1 copy for every 2 students): Activity 1
12	• Geometry toolkits: Activity 1	• Find Someone Similar Cards (1 copy for every 10 students): Activity 3

	13	• Blank paper: Activity 1 • Dried linguine pasta: Activity 1 • Geometry toolkits: Activity 1, Activity 2 • Protractors: Activity 1 • Rulers: Activity 1 • Tape: Activity 1	• Making Pasta Angles and Triangles Cards (1 copy for every 4 students): Activity 1
	14	• Geometry toolkits: Warm-up	
	15	• Geometry toolkits: Warm-up • Tracing paper: Warm-up • Straightedges: Activity 2	
	16	• Rulers: Warm-up • Straightedges: Warm-up	
	17	• Geometry toolkits: Warm-up, Activity 1, Activity 2 • Straightedges: Activity 2	
	18	• Blank paper: Activity 1 • Measuring tools: Activity 1 • Graph paper: Activity 2	
	19	• Measuring tapes: Activity 2, Activity 3 • Rulers: Activity 2, Activity 3 • Yardsticks: Activity 2, Activity 3	

Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions:**
Astronomy: Students use unitless scales to calculate the vast distances between the sun and various planets	**Slope Calculation:** Students may divide the horizontal change by the vertical change (x/y) instead of y/x See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:	**Connections to Future Units:**
Essential prior concepts to engage with this unit: • Fractions as numbers (Grades 3–5) • Equivalent fractions (Grade 4–5) • Ratios and unit rates (Grade 6) • Whole number and decimal multiplication/division (Grades 4–6) • Angle measurement (Grade 4) • Measuring lengths (Grades K–5) • Graphing points and finding distances (Grade 5) • Area formulas for rectangles, triangles (Grades 3–5) **Relevant Unit(s)/Lesson(s) to Review:** • Fractions and equivalent fractions → Ratios & unit rates → Multiplying by scale factors. If students lack ratio fluency, they will struggle with the entire unit.	Transitions students from dilations and similarity to understanding linear relationships
Differentiation through *Universal Design for Learning*	

Engagement:

- Provide access to pre-cut materials like rectangles to reduce barriers for students with fine-motor skill needs (Lesson 8, Activity 1 Launch)

LT5: Create a dilation of a figure given a scale factor and center

Representation:

- Use multiple examples and non-examples to reinforce the differences between similar and non-similar polygons (Lesson 12, Activity 3 Launch)

LT9: Justify that two triangles are similar by finding a sequence of transformations

Action & Expression:

- Invite students to create stories for equations that connect to their own lives (Lesson 6, Activity 2 Student Task Statement)

LT4: Create a scale drawing given actual measurements or another scale drawing

Supporting Multilingual Learners

Math Language Routines

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MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
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MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 7, 12, 15, 19)
- MLR7: Compare and Connect (Lessons 5, 7, 10, 13, 14, 17)
- MLR8: Discussion Supports (Lessons 1, 2, 3, 4, 5, 12, 13, 15, 16, 18)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, explaining, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sensemaking and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Features of scaled copies (Lesson 1 and 2).
- Observations about scaled rectangles (Lesson 8).
- Observations about dilated points, circles, and polygons (Lesson 9).
- Sequences of transformations (Lesson 12).
- Observations about side lengths in similar triangles (Lesson 14).

Explain

- How to use scale drawings to find actual distances (Lesson 4 and 7).
- How to apply dilations to find specific images (Lesson 10).
- How to determine whether triangles are congruent, similar, or neither (Lesson 13).
- Strategies for finding missing side lengths (Lesson 14).
- How to apply dilations to find specific images of points (Lesson 17).
- Reasoning for a conjecture (Lesson 19).

Represent

- A scaled copy for a given scale factor (Lessons 2 and 3).
- Distances using different scales (Lesson 7).
- Dilations using given scale factors and coordinates (Lesson 10).
- Figures using specific transformations (Lesson 12).
- Graphs of lines using equations (Lesson 17).
- Relevant features of a classroom with a scale drawing (Lesson 18).

Sentence Frames and Stems

Section A

- I know _____ is/is not a scaled copy of _____ because ...
- Corresponding side lengths in scaled copies _____ . Angle measures in scaled copies _____.
- The scale factor between _____ and _____ is _____ because ...
- To create a scaled copy of the figure _____ , I multiplied each side length by the scale factor _____ to get side lengths ...
- If the scale factor is greater/less than one, the copy will be _____ because ...

Section B

- The scale shows that _____ on the drawing represents _____ on the actual object.
- Since the scale is _____ to _____ , then _____ on the drawing means _____ on the actual object.
- When a scale drawing has a scale of _____ to _____ with no units, it means ...
- The scale _____ to _____ and _____ to _____ are equivalent because ...
- If the scale of the drawing is _____, the area of the actual object would be _____ because...

Section C

- To create a dilation of figure _____ with center _____, first I _____, then I ...
- I know the scale factor is _____ because ...
- I know the center of dilation is _____ because ...
- The image of figure _____ was dilated with a scale factor of _____ and a center _____. The coordinates of the image are ...
- I used _____ to dilate figure _____ because ...

Section D

- The ratio of side lengths _____ and _____ is equivalent to the ratio of the corresponding side lengths _____ and _____. This means ...
- The transformations ... move figure _____ to figure _____. The figures are similar because ...
- Figure _____ is similar to figure _____ because ...
- Triangle _____ is similar to triangle _____ because ...

Section E

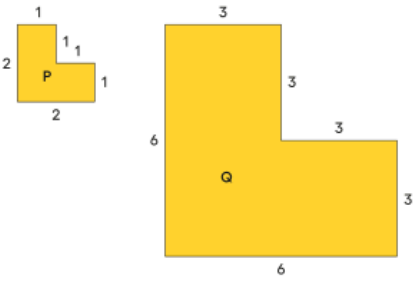
- The slope of a line is a value that describes ...
- I know that line _____ has a slope of _____ because ...
- To draw a line with a slope of _____, first I _____, then I ...
- The point _____ is on the line because the equation for the line is _____ and ...

Section F

- I chose to use a scale of _____ to _____ because ...
- Since the length of _____ is _____ and I want to make a drawing with length of _____, my scale will be _____ to _____.
- To make a scale floor plan of _____, first I would ...
- If I know the shadow length of the _____ is _____, then the height of the lamppost is _____ because ...
- The triangles created by the objects and their shadows are similar because ...

Unit Outline

In this unit, students study scaled copies of plane figures and scale drawings of real-world objects. Students learn that all lengths in a scaled copy are the result of multiplying the original lengths by a scale factor.
Also, the angle measures in a scaled copy are the same as in the original figure.



This work builds on what students learned in previous grades about measuring lengths, areas, and angles. This unit provides a geometric context to preview the type of reasoning that students will use with proportional relationships and also lays the foundation for work on dilations and similarity.

Students begin the unit by looking at copies of a picture and describing what differentiates scaled and non-scaled copies. They calculate scale factors and draw scaled copies of figures.

Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can also be used with scale drawings. They use scale drawings to calculate actual lengths and areas, and they create scale drawings.

In the next two sections, students learn about dilations as a new transformation that creates scaled copies. They connect dilations to earlier work with rigid transformations as they explain why two figures are similar by describing a sequence of translations, reflections, rotations, and dilations that take one figure to the other. They discover that angle measures in similar figures are preserved, which can be used to justify that two triangles are similar if they share two (or three) angle measures. Students also find that the quotients of corresponding side lengths in similar figures are equal. This along with the fact that side lengths in similar figures are all multiplied by the same scale factor allows students to calculate unknown lengths in similar figures.

In the following section, students use the similarity of slope triangles to understand why any two distinct points on a line determine the same slope. Using these same properties of similar triangles, students practice writing equations for a given line, though students are not expected at this time to write equations in the form y=mx+b.

In this unit, several lesson plans suggest that each student have access to a geometry toolkit. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems. Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment

Section A Scaled Copies (Lessons 1-3)	**Learning Target #1:** Determine whether a figure is a scaled copy of another figure, by examining corresponding side lengths and angle measures. **Learning Target #2** Draw a scaled copy of a figure using a given scale factor.	**Lesson 1 What Are Scaled Copies?** • I can describe some characteristics of a scaled copy. • I can tell whether or not a figure is a scaled copy of another figure **Lesson 2 Scale Factors and Making Scaled Copies** • I can describe what the scale factor has to do with a figure and its scaled copy. • I can draw a scaled copy of a figure using a given scale factor. • I know what operation to use on the side lengths of a figure to produce a scaled copy. **Lesson 3 Scaled Relationships** • I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1. • I can explain how the scale factor that takes Figure A to its copy Figure B is related to the scale factor that takes Figure B to Figure • I can use corresponding distances and corresponding angles to te whether one figure is a scaled copy of another.
Checkpoint A	***Responding to Student Thinking*** • **Problem 1:** More Chances: Students will have more opportunities to develop this understanding in la lessons. There is no need to slow down or add additional work to review this concept at this time. • **Problem 2:** Points to Emphasize: If most students struggle with creating a scaled copy of a figure, revisit the characteristics of scaled copies when opportunities arise over the next several lessons. Fo example, invite multiple students to share their thinking about how to create a scale drawing in these activities: ○ Accelerated 7, Unit 2, Lesson 5, Activity 2 Two Maps of Utah ○ Accelerated 7, Unit 2, Lesson 6, Activity 2 A New Drawing of the Playground	
Section B Scale Drawings (Lessons 4-7)	**Learning Target #3** Create a scale drawing given the actual measurements of the object or given another scale drawing at a different scale. **Learning Target #4** Explain how to use scales and scale drawings to calculate actual distances and areas.	**Lesson 4 Scale Drawings** • I can explain what a scale drawing is, and I can explain what its sca means. • I can use actual distances and a scale to find scaled distances. • I can use a scale drawing and its scale to find actual distances. **Lesson 5 Creating Scale Drawings** • I can determine the scale of a scale drawing when I know lengths the drawing and corresponding actual lengths. • I know how different scales affect the lengths in the scale drawin • When I know the actual measurements, I can create a scale drawi at a given scale. **Lesson 6 Changing Scales in Scale Drawings** • Given a scale drawing, I can create another scale drawing that sho the same thing at a different scale. • I can use a scale drawing to find actual areas. **Lesson 7 Units in Scale Drawings** • I can use scales without units to find scaled distances or actual distances.
Checkpoint B	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of calculating actual lengths and areas from a scale drawing. If students struggle with this, make time to examine relat work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier wo ○ Accelerated 7, Unit 2, Section B Scale Drawings	
Section C Dilations (Lessons 7-11)	**Learning Target #5** Create a dilation of a figure given a scale factor and center of dilation. **Learning Target #6** Describe a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points. **Learning Target #7** Identify the center, scale factor, and image of a dilation.	**Lesson 8 Projecting and Scaling** • I can decide if one rectangle is a scaled copy of another rectangle **Lesson 9 Dilations** • I can apply a dilation to a polygon using a ruler. • I can apply dilations to figures on a circular grid when the center o dilation is the center of the grid. **Lesson 10 Dilations on a Square Grid** • I can apply dilations to figures on a square grid. **Lesson 11 More Dilations** • I can apply dilations to polygons on a rectangular grid if I know the coordinates of the vertices and of the center of dilation.

<table>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
● Problem 1: Points to Emphasize: If most students struggle with finding the center of dilation, revisit how describe dilations when showing that two figures are similar. For example, in the activity referred to here emphasize the location of the center of the dilation.
○ Accelerated 7, Unit 2, Lesson 12, Activity 1 Similarity Transformations
● Problem 2: Points to Emphasize: If most students struggle with drawing a dilation on the coordinate pla revisit how to do this before starting the activity referred to here.
○ Accelerated 7, Unit 2, Lesson 17, Activity 1 Dilations and Slope Triangles</td></tr>
<tr><td>Section D
Similarity
(Lessons 12-16)</td><td>Learning Target #8
Calculate unknown side lengths in similar triangles using the ratios of side lengths within the triangles and the scale factor between similar triangles.

Learning Target #9
Justify that two triangles are similar by finding a sequence of transformations that takes one triangle to the other or by checking that two pairs of corresponding angles are congruent.</td><td>Lesson 12 Similarity
● I can use angle measures and side lengths to conclude that two polygons are not similar.
● I can use a sequence of transformations to explain why two figure are similar.
Lesson 13 Similar Triangles
● I know how to decide if two triangles are similar just by looking at their angle measures.
Lesson 14 Side Length Quotients in Similar Triangles
● I can decide if two triangles are similar by looking at quotients of lengths of corresponding sides.
● I can find missing side lengths in a pair of similar triangles using quotients of side lengths.</td></tr>
<tr><td>Checkpoint D</td><td colspan="2">Responding to Student Thinking
● Problem 1: Points to Emphasize: If most students struggle with explaining why two figures are similar, before starting the activity referred to here, emphasize the different ways to show similarity.
○ Accelerated 7, Unit 2, Lesson 15, Activity 1 Similar Triangles on the Same Line
● Problem 2: More chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.</td></tr>
<tr><td>Section E
Slope
(Lessons 17-18)</td><td>Learning Target #10
Comprehend the term “slope” to mean a number that tells how steep a line is.

Learning Target #11
Create an equation relating the quotient of the vertical and horizontal side lengths of a slope triangle to the slope of a line and use it to justify whether a point (x,y) is on the line by verifying that the values of x and y satisfy the equation.</td><td>Lesson 15 Meet Slope
● I can draw a line on a grid with a given slope.
● I can find the slope of a line on a grid.
Lesson 16 Writing Equations for Lines
● I can decide whether a point is on a line by finding quotients of horizontal and vertical distances.
Lesson 17 Using Equations for Lines
● I can find an equation for a line and use it to decide which points a on that line.</td></tr>
<tr><td>Checkpoint E</td><td colspan="2">Responding to Student Thinking
● Problem 1: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.
● Problem 2: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.</td></tr>
<tr><td>Section F
Let’s Put
it To Work
(Lesson 19)</td><td>No new learning targets</td><td>Lesson 18 Draw It to Scale
● I can create a scale drawing of my classroom.
● When given requirements on drawing size, I can choose an appropriate scale to represent an actual object.
Lesson 19 The Shadow Knows
● I can model a real-world context with similar triangles to find the height of an unknown object</td></tr>
<tr><td colspan="3">End of Unit Assessment
● Combine mid unit and EOU
○ Mid Unit: 1,2,3,5
○ EOU: 1,3,4,7</td></tr>
</table>

Unit Title:

Unit 8: Linear Relationships (G7 ACC U5)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	8.EE.B	Lesson 15	8.EE.C.8, 8.EE.C.8.a
Lesson 2	8.EE.b, 8.EE.B.5	Lesson 16	8.EE.C.8
Lesson 3	8.EE.b, 8.EE.B.5	Lesson 17	8.EE.C.8, 8.EE.C.8.b, 8.EE.C.8.c
Lesson 4	8.EE.B	Lesson 18	8.SP.A.1
Lesson 5	8.EE.b, 8.EE.B.5	Lesson 19	8.SP.A.1, 8.SP.A.3
Lesson 6	8.EE.b, 8.EE.B.6	Lesson 20	8.SP.A.1, 8.SP.A.2
Lesson 7	8.EE.B	Lesson 21	8.SP.A.1, 8.SP.A.2
Lesson 8	8.EE.B	Lesson 22	8.SP.A.1, 8.SP.A.2, 8.SP.A.3
Lesson 9	8.EE.b, 8.EE.B.5	Lesson 23	8.SP.A.1, 8.SP.A.2, 8.SP.A.3
Lesson 10	8.EE.b, 8.EE.B.6	Lesson 24	8.SP.A.4
Lesson 11	8.EE.b	Lesson 25	8.SP.A.4
Lesson 12	8.EE.C	Lesson 26	8.EE.B.6, 8.EE.C.8.a
Lesson 13	8.EE.C.8	Lesson 27	8.EE.C.8.c
Lesson 14	8.EE.C.8, 8.EE.C.8.a, 8.EE.C.8.b	Lesson 28	8.SP.A

Essential Question(s):

- What defines a linear relationship and how does it differ from a proportional one?
- How do the slope and y-intercept of a line communicate the story of a real-world situation?
- How can different mathematical representations (tables, equations, graphs) be used to analyze and predict data?
- What does the intersection of two lines represent in a system of equations?

Enduring Understanding(s):

- A linear relationship is characterized by a constant rate of change between two quantities. While all proportional relationships are linear, a linear relationship is only proportional if its graph passes through the origin (0, 0)
- In a linear context, the slope represents the rate of change (how much the dependent variable changes for every unit increase in the independent variable), and the y-intercept represents the initial or starting value
- Linear relationships can be modeled using equations in forms like y=mx+b or Ax+By=C. These models allow for the prediction of unknown values and help determine if specific coordinate pairs represent valid solutions to a given constraint
- A system of equations models two or more relationships happening at once; the point where the lines intersect is the unique solution that satisfies both relationships simultaneously. Systems may also have no solution (parallel lines) or infinitely many solutions (identical lines)

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 3)
CFA 2: Checkpoint B (after lesson 7)
CFA 3: Checkpoint C (after lesson 12)
MoU: Assessment A (after lesson 12)
CFA 4: Checkpoint D (after lesson 17)
EoU: Assessment A (after lesson 17)
*omit questions 6&7 (data), added 3 questions from a previous version of the assessment

Pacing for Unit

25 Days
Lesson Modifications:

- 10 flex days: Based on student need, consider:
  - Optional lessons: 6 (Lessons 10, 24, 25, 26, 27, and 28)
  - Omit Lessons 1-2 (proportional relationships)
  - Omit Lesson 4 (intro to linear relationships)
  - Omit Lessons 18-25 (data)
- 4 days for review/assessment

Family Overview

https://accessim.org/6-8-accelerated/accelerated-7/unit

Integration of Technology:

- Desmos Online Graphing Calculator

-5?a=family

- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology: receptive	New Terminology: productive
Acc7.5.1	represent scale label	constant of proportionality
Acc7.5.2	rate of change equation	
Acc7.5.4	linear relationship constant rate rate of change	slope
Acc7.5.5	vertical intercept y-intercept	
Acc7.5.6	initial (value or amount)	constant rate
Acc7.5.7	relate	
Acc7.5.8	horizontal intercept x-intercept	
Acc7.5.9		rate of change vertical intercept y-intercept
Acc7.5.10	constraint	horizontal line vertical line
Acc7.5.11	solution to an equation with two variables variable combination set of solutions	
Acc7.5.13	ordered pair	
Acc7.5.14	system of equations solution to a system of equations	
Acc7.5.15	substitution	no solution (only) one solution infinitely many solutions
Acc7.5.15	algebraically	
Acc7.5.17		system of equations substitution
Acc7.5.18	scatter plot	
Acc7.5.18	attribute input output	numerical data
Acc7.5.20	outlier predict overpredict underpredict linear model	
Acc7.5.21	positive association negative association	
Acc7.5.22	linear association non-linear association no association	

Aligned Unit Materials, Resources, and Technology

Digital Applets

- 5.6 Rising Water Levels
- 5.7 Increased Savings, Translating a Line
- 5.8 Stand Clear of Closing Doors, Please, Travel Habits in July
- 5.14 Passing on the Trail, Stacks of Cups
- 5.15 Matching Graphs to Systems, Different Types of Systems
- 5.18 Weight and Fuel Efficiency
- 5.20 Battery Life, The Agony of the Feet
- 5.21 Fitting Lines

Provide access as needed throughout the unit:

- Colored pencils
- Dried linguine pasta (We specified linguine since it is flatter and less likely to roll around than spaghetti.)
- Geometry toolkits
- Graduated cylinders
- Graph paper
- Math Community Chart
- Scissors
- Stopwatches
- StraightedgesTeacher's collection of objects
- Tools for creating a visual display
- Water

Lesson	Materials to Gather	Materials to Copy
2	• Straightedges: Activity 2, Activity 3	• Proportional Relationship Cards (1 copy for every 4 students): Activity 1 • Graphing Proportional Relationships Cards (1 copy for every 2 students): Activity 4
3	• Math Community Chart: Activity 1 • Tools for creating a visual display: Activity 1	
4	• Graph paper: Activity 1 • Straightedges: Activity 1, Activity 2	
5		• Slopes, Vertical Intercepts, and Graphs Cards (1 copy for every 2 students): Activity 1
6	• Graduated cylinders: Activity 1 • Straightedges: Activity 1 • Teacher's collection of objects: Activity 1 • Water: Activity 1	
7	• Geometry toolkits: Warm-up	• Translating a Line Cards (1 copy for

	fitted line	
Acc7.5.23	cluster	positive association negative association linear association
Acc7.5.24	segmented bar graph relative frequency two-way (frequency) table	
Acc7.5.28		scatter plot outlier cluster

		every 2 students): Activity 2
8	• Straightedges: Activity 1	
9	• Straightedges: Activity 3	• Making Designs Cards (1 copy for every 2 students): Activity 3
10	• Straightedges: Activity 1, Activity 2	
11	• Colored pencils: Activity 2 • Graph paper: Activity 2 • Straightedges: Activity 2	
12		• I'll Take an X Please Cards (1 copy for every 2 students): Activity 2
13	• Straightedges: Lesson	
14	• Straightedges: Activity 1, Activity 2	
15	• Scissors: Activity 2 • Straightedges: Activity 2	• Different Types of Systems Handout (1 copy for every 2 students): Activity 2
17		• Racing and Play Tickets Cards (1 copy for every 4 students): Activity 1
18		• Tables and Their Scatter Plots Handout (1 copy for every 2 students): Activity 2
21	• Dried linguine pasta: Activity 1 • Straightedges: Activity 1	
22		
23	• Dried linguine pasta: Activity 2 • Straightedges: Activity 2	• Scatterplot City Cards (1 copy for each student): Activity 1
24		• Matching Representations Cards (1 copy for every 2 students): Activity 1
25	• Colored pencils: Activity 2 • Straightedges: Activity 2	
27	• Tools for creating a visual display: Activity 1	
28	• Stopwatches: Warm-up	

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
Biology: Linear models compare the growth rates of different plants over	**Steepness vs. Slope:** Students may judge a line's steepness visually without checking the scale of the axes See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:	**Connections to Future Units:**
Essential prior concepts to engage with this unit: • Ratios and proportional relationships (Grade 6) • Unit rates and the constant of proportionality (Grade 6, Unit 2) • Interpreting graphs and tables (Grade 6) • Plotting points and reading coordinates (Grade 5–6 • Finding distances on coordinate planes (Grade 5–6) • The coordinate plane as a representation of relationships (Grade 6) • Solving linear equations (Grade 7 Unit 3–4) • Writing equations in various forms (write a linear equation given a point and slope or given two points) • Understanding negative slopes (6ACC Unit 7) • Operations on rational numbers • Recognizing patterns in tables & graphs (6ACC Unit 5) • Estimating and predicting using trends **Relevant Unit(s)/Lesson(s) to Review:** • Spiral review of proportional relationships (using Unit 2 or Grade 6 content), solving equations (Units 3–4), and coordinate graphing (Grade 5–6) into early lessons. • Students often need multiple exposures to slope concepts before grasping rate of change, especially when negative numbers are involved.	Advances understanding of lines for use in scatter plots and fitted lines to analyze numerical data. Lays foundation for functions in Unit 9

Differentiation through *Universal Design for Learning*

Engagement:
- Provide an opportunity for students to self-assess their confidence that their description matches a chosen line (Lesson 26, Activity 1 Launch)
  LT5: Interpret the slope and intercept of the graph of a line in context

Representation:
- Maintain a display of vocabulary and diagrams for width, length, perimeter, and intercept (Lesson 10, Activity 2 Launch)
  LT7: Determine pairs of values that satisfy or do not satisfy a linear relationship

Action & Expression:
- Use sticky notes for working memory support during Math Talks (Lesson 9, Warm-up Launch)
  LT6: Create multiple representations of a linear relationship

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:
- MLR5: Co-Craft Questions (Lessons 1, 4, 15, 19)
- MLR6: Three Reads (Lessons 2, 3, 5, 13, 14)
- MLR8: Discussion Supports (Lessons 2, 5, 7, 9, 11, 16, 17, 23, 26, 27)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as representing, interpreting, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sensemaking and for building shared understanding with peers. Teachers

can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- Situations involving proportional relationships (Lesson 1).
- Constants of proportionality in different ways (Lesson 2).
- Linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 4).
- Situations using negative slopes and slopes of zero (Lesson 8).
- Slope using expressions (Lesson 9).
- Situations by graphing lines and writing equations (Lesson 11).
- Situations involving systems of linear equations. (Lessons 13, 14, and 16).
- Data in organized ways (Lesson 18).
- Data using two-way tables, bar graphs, and segmented bar graphs (Lessons 24 and 25).
- Data using scatter plots (Lesson 28).

Interpret

- Situations involving proportional relationships (Lesson 1).
- Slopes and intercepts of linear graphs (Lesson 2).
- Situations using negative slopes and slopes of zero (Lesson 9).
- Situations involving systems of linear equations (Lessons 13 and 14).
- Tables and scatter plots of bivariate data (Lesson 19).
- Tables, scatter plots, equations, and situations involving bivariate data (Lesson 20).
- Situations involving linear relationships (Lesson 26).

Explain

- How to graph proportional relationships (Lesson 2).
- How to use a graph to determine information about a linear situation (Lessons 4 and 5).
- How to graph linear relationships (Lesson 9).
- How to estimate using available data (Lesson 18).
- How to use tables and scatter plots to make estimates and predictions (Lesson 19).
- The meaning of slope for a situation (Lesson 20).
- How to use lines to show associations, identify outliers, and answer questions (Lesson 23).
- How to answer questions about systems of equations (Lesson 27).

Sentence Frames and Stems

Section A

- The relationship between _____ and _____ is proportional because ...
- The equation _____ represents this proportional relationship because ...
- To create a graph of a proportional relationship, first I _____, then I ...
- I can substitute the value _____ into the equation _____ to find the value of _____.

Section B

- The graphs of the lines with equations _____ and _____ are/aren't parallel. I know because in the equations ...
- The slope of the line is _____ because ...
- The slope in this situation is _____ and represents _____.
- The y-intercept of the line is _____ and represents _____.
- The equation _____ represents this situation because ...

Section C

- I used _____ to represent the linear relationship between _____ and _____ because ...
- The slope in this situation is _____ and represents _____.
- To draw a line with a slope _____ and the point _____ on the line, first I _____, then I ...
- The equation _____ describes the line because ...
- The points ... are solutions to the equation _____ because ...
- The pair of values _____ do/do not satisfy the equation _____ because ...

Section D

- The system of equations has _____ solution(s). I know this because ...
- The solution to the system of equations is _____ because ...
- This situation can be represented by the system of equations _____ because ...
- To solve the system of equations, first I _____, then I ...

Section E

- Each point on the scatter plot represents _____ and _____.
- The data in this scatter plot represent ...
- I can locate a specific data point on a scatter plot by ...
- The association between _____ and _____ is _____ because ...
- The outlier in the data set is _____ because ...
- The linear equation _____ would be a good fit for this data because ...
- If the _____ increases by 1 _____ , the model predicts that _____ increases/decreases by _____ .
- To draw a line that best fits the data, first I _____, then I ...

Section F

- Using the data shown in the _____, I can determine that ...
- To find the missing values in the table, first I _____, then I ...
- The percentage of _____ that _____ is _____ because ...
- I created a _____ to show the association between _____ and _____ is _____.

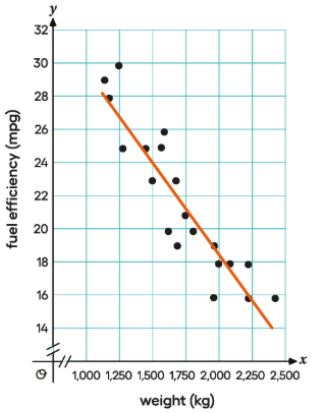
Section G

- The equation _____ represents this situation because ...
- The account starts at _____ and increases/decreases by _____ per _____.
- This situation can be represented by the system of equations _____ because ...
- To solve the system of equations, first I _____, then I ...
- The solution to the system of equations is _____ because ...
- Comparing Time 1 to Time 2, there is a _____ association because ...
- The _____ was most helpful to determine if there was an association because ...

Unit Outline

This unit introduces students to nonproportional linear relationships by building on earlier work around similarity and slope. Then students solve systems of linear equations using graphic and algebraic methods. Students advance their understanding of lines by examining slopes in the context of data. Lastly, they use scatter plots and fitted lines to analyze numerical data.

The unit begins by revisiting different representations of proportional relationships. Students create graphs, tables, and equations in order to interpret the constant of proportionality as the rate of change of one variable with respect to the other.



Next, students analyze a relationship that is linear but not proportional. They see that the rate of change has a numerical value that is the same as the slope of the line that represents the relationship. Students also view the graph of a line in the coordinate plane as the vertical translation of a proportional relationship.

Then students consider situations represented by linear relationships with negative rates of change. They establish a way to compute the slope of a line from any two distinct points on the line.

Next students examine systems of equations graphically and find solutions algebraically. They build on their understanding that the line representing an equation with 2 variables is made up of coordinate pairs that make the equation true. They find that the intersection of 2 lines is the point that makes both equations for the system true. Students also recognize when systems have no solution or infinite solutions based on the graphs and the slope and intercept.

Then students are introduced to scatter plots and are reminded how to interpret points on a graph using a context. They look more closely at associations in data by informally drawing lines that model the general trend of the data. They also classify associations as positive, negative, linear, and non-linear by looking at the shape of the data in a scatter plot.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Proportional Relationships (Lessons 1-3)	**Learning Target #1:** Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes. **Learning Target #2** Interpret multiple representations of a proportional relationship in context.	**Lesson 1 Understanding Proportional Relationships** • I can graph a proportional relationship from a story. • I can use the constant of proportionality to compare the pace of different animals. **Lesson 2 Representing Proportional Relationships** • I can scale and label coordinate axes in order to graph a proportio relationship. • I can tell when two graphs are of the same proportional relationsh even if the scales are different. **Lesson 3 Comparing Proportional Relationships** • I can compare proportional relationships represented in different ways.
Checkpoint A	***Responding to Student Thinking*** • **Problem 1 & 2:** More Chances: Students will have more opportunities to develop this understanding later lessons. There is no need to slow down or add additional work to review this concept at this tim	
Section B Linear Equations in One Variable (Lessons 4-7)	**Learning Target #3** Create and compare graphs that represent linear relationships with the same rate of change but different initial values.	**Lesson 4 Introduction to Linear Relationships** • I can find the rate of change of a linear relationship by figuring ou the slope of the line representing the relationship. **Lesson 5 More Linear Relationships** • I can interpret the vertical intercept of a graph of a real-world situation.

	Learning Target #4 Create an equation that represents a linear relationship. **Learning Target #5** Interpret the slope and intercept of the graph of a line in context.	• I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept. **Lesson 6 Representations of Linear Relationships** • I can use patterns to write a linear equation to represent a situatio • I can write an equation for the relationship between the total volu in a graduated cylinder and the number of objects added to the graduated cylinder. **Lesson 7 Translating to** • I can explain where to find the slope and vertical intercept in both equation and its graph. • I can write equations of lines using .
Checkpoint B	***Responding to Student Thinking*** • **Problem 1:** Points to Emphasize: If most students struggle with interpreting the slope of a line in contex revisit the concept of rate of change. For example, in the section referred to here, discuss the similarity between graphing and interpreting negative rates of change and positive rates of change. ○ Accelerated 7, Unit 5, Section C Finding Slopes and Linear Equations	
Section C Finding Slopes (Lessons 8-12)	**Learning Target #6** Create multiple representations of a linear relationship, including a graph, equation, and table. **Learning Target #7** Determine pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph. **Learning Target #8** Interpret the slope of a non-increasing line in context.	**Lesson 8 Slopes Don't Have to Be Positive** • I can create a graph of a situation that has a negative slope. • I can determine if a situation or a graph has a slope that is positive negative, or zero and explain how I know. **Lesson 9 Calculating Slope** • I can calculate positive and negative slopes given two points on t line. **Lesson 10 Equations of All Kinds of Lines** • I can write equations of lines that have a positive or a negative slo • I can write equations of vertical and horizontal lines. **Lesson 11 Solutions to Linear Equations** • I know that the graph of an equation is a visual representation of the solutions to the equation. • I understand what the solution to an equation in two variables is. **Lesson 12 More Solutions to Linear Equations** • I can find solutions to linear equations given either the - or -value start from.
Checkpoint C	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of interpreting slope in context. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work. ○ Accelerated 7, Unit 5, Section B Representing Linear Relationships • **Problem 2:** Points to Emphasize: If most students struggle with determining whether or not a pair of values satisfies a linear relationship, revisit the concept of what it means to be a solution to an equation For example, in the activity referred to here, emphasize how to check that the coordinates of the point intersection of two lines satisfies the equations of both lines. ○ Accelerated 7, Unit 5, Section B Representing Linear Relationships	
Mid-Unit Assessment		
Section D Systems of Linear Equations (Lessons 13-17)	**Learning Target #9** Categorize systems of equations, including systems with infinitely many or no solutions, and calculate the solution for a system using a variety of strategies. **Learning Target #10** Comprehend that solving a system of equations means finding values of the variables that make both equations true at the same time. **Learning Target #11** Create a system of equations that represents a situation and interpret the solution in context.	**Lesson 13 On Both of the Lines** • I can use graphs to find an ordered pair that two real-world situations have in common. **Lesson 14 Systems of Equations** • I can explain the solution to a system of equations in a real-world context. • I can explain what a system of equations is. • I can make graphs to find an ordered pair that two real-world situations have in common. **Lesson 15 Solving Systems of Equations** • I can graph a system of equations. • I can solve systems of equations using algebra. **Lesson 16 Solving More Systems** • I can use the structure of equations to help me figure out how ma solutions a system of equations has. **Lesson 17 Writing Systems of Equations** • I can write a system of equations from a real-world situation.

Checkpoint D	***Responding to Student Thinking*** ● **Problem 1:** Press Pause: If students struggle to classify the number of solutions for a system, make tim for students to revisit what a solution for a system means for each equation, in any situation given, and graphically. For example, revisit the situation with bug passing. Ask students to write a system for the situation and to interpret the intersection point in all 3 ways. Then, ask students what it might look like i the bugs never passed or were together the entire time. ○ Accelerated 7, Unit 5, Lesson 13, Activity 1 Bugs Passing in the Night ● **Problem 2:** Points to Emphasize: If students struggle to write a system of equations to represent a situation, move more slowly through the practice problems and activities related to the idea. For examp in this activity, ask students to identify the variables and constants involved and then to combine them i an equation for each person in the scenarios: ○ Accelerated 7, Unit 5, Lesson 27, Activity 1 Cycling, Fundraising, Working, and ____?	
Section E Does This Predict That? (Lessons 18-23)	**Learning Target #12** Create a scatter plot from a table of data, and describe the trend of the data. **Learning Target #13** Describe the relationship between two variables using a line fit to data on a scatter plot. **Learning Target #14** Interpret features of data on a scatter plot, including linear and non-linear association, outliers, slope of a linear model, and clustering.	**Lesson 18 Organizing Data** ● I can organize data to see patterns more clearly. **Lesson 19 What a Point in a Scatter Plot Means** ● I can describe the meaning of a point in a scatter plot in context. **Lesson 20 Fitting a Line to Data** ● I can pick out outliers on a scatter plot. ● I can use a model to predict values for data. **Lesson 21 Describing Trends in Scatter Plots** ● I can draw a line to fit data in a scatter plot. ● I can say whether data in a scatter plot has a positive or negative association (or neither). **Lesson 22 The Slope of a Fitted Line** ● I can use the slope of a line fit to data in a scatter plot to say how t variables are connected in real-world situations. **Lesson 23 Observing More Patterns in Scatter Plots** ● I can pick out clusters in data from a scatter plot. ● I can use a scatter plot to decide if two variables have a linear association.
Checkpoint E	***Responding to Student Thinking*** ● **Problem 1:** More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. ● **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of slope an its meaning. If students struggle, make time to revisit related work in the referenced lesson. See the Course Guide for ideas to help students re-engage with earlier work. For example, revisit contexts fr linear situations throughout this course and ask students to determine the slope and interpret its meaning. ○ Accelerated 7, Unit 5, Lesson 22 The Slope of a Fitted Line ● **Problem 3:** More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.	
Section F Associations in Categorical Data (Lesson 24-25	**Learning Target #15** Calculate relative frequencies, and describe associations between variables using a relative frequency table. **Learning Target #16** Create a two-way table and a segmented bar graph that represent relative frequencies, and interpret the frequencies in context.	**Lesson 24 Looking for Associations** ● I can identify the same data represented in a bar graph, a segmen bar graph, and a two-way table. ● I can use a two-way frequency table or relative frequency table to find associations among variables. **Lesson 25 Using Data Displays to Find Associations** ● I can create relative frequency tables, bar graphs, and segmented bar graphs from frequency tables to find associations among variables.
Checkpoint F	***Responding to Student Thinking*** **Problem 1:** More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.	
Section G Let's Put it To Work (Lesson 26-28)	**No new learning targets**	**Lesson 26 Using Linear Relations to Solve Problems** ● I can write linear equations to reason about real-world situations. **Lesson 27 Solving Problems with Systems of Equations** ● I can use a system of equations to represent a real-world situatio and answer questions about the situation. **Lesson 28 Gone in 30 Seconds**

		• I can collect data and analyze it for associations using scatter plo two-way tables, and segmented bar graphs
End of Unit Assessment		

Unit Title:

Unit 9: Functions and Volume (G7 ACC U6)

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	8.F.A.1	Lesson 13	7.G.B.6
Lesson 2	8.F.A.1	Lesson 14	7.G.B, 7.G.B.6, 7.RP.A
Lesson 3	8.F.a, 8.F.A.1	Lesson 15	8.G.C.9
Lesson 4	8.F.A.1, 8.F.A.3, 8.F.B.5	Lesson 16	8.G.C.9
Lesson 5	8.F.B.4, 8.F.B.5	Lesson 17	8.F.A.1, 8.F.A.3, 8.F.b
Lesson 6	8.F.A.2, 8.F.A.3	Lesson 18	8.F.A.3, 8.F.b, 8.G.C.9, 8.G.C, 8.G.C.9
Lesson 7	8.F.A.2, 8.F.A.3, 8.F.B.4	Lesson 19	8.G.C, 8.G.C.9
Lesson 8	7.G.A.3, 8.F.B.4, 8.F.B.5	Lesson 20	8.G.C, 8.G.C.9
Lesson 9	7.G.B.6	Lesson 21	8.G.C.9
Lesson 10	7.G.A.3, 7.G.B.6	Lesson 22	7.G.A.2, 7.G.B.6
Lesson 11	8.G.C.9	Lesson 23	8.F.a, 8.G.C.9
Lesson 12	8.G.C.9		

Essential Question(s):

- What is a mathematical function, and how is it represented in different ways?
- How are the volume formulas for cylinders, cones, and spheres related?
- How does changing dimensions affect the volume of a three-dimensional object?

Enduring Understanding(s):

- A function is a relationship where each input determines exactly one output; it can be represented by a rule, a table, a graph, or an equation
- The volumes of cylinders, cones, and spheres with the same radius and height follow a 3:1:2 ratio; for example, a cone's volume is exactly ⅓ of a cylinder with the same dimensions
- Volume is a function of a solid's dimensions; while scaling height results in a proportional change in volume, scaling the radius results in a non-proportional change because the radius is squared or cubed in the volume formula

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 7)
MoU: Assessment A (after lesson 7)
CFA 2: Checkpoint B (after lesson 14)
CFA 3: Checkpoint C (after lesson 21)
EoU: Assessment A (after lesson 21)

Pacing for Unit

17 days
Lesson Modifications:

- Covering Lessons 1-21
- Combine Lessons 1&2, 3&4
- Omit Lesson 5 (optional)
- Optional lessons: 5 (Lessons 14, 17, 18, 22, and 23)
- 4 days for review/assessment
- 3 flex days

Family Overview

https://accessim.org/6-8-accelerated/accelerated-7/unit-6?a=family

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
Acc7.6.1	input output	
Acc7.6.2	function	input output depends on
Acc7.6.3	independent variable	

Aligned Unit Materials, Resources, and Technology

Digital Applets

- 6.1 Guess My Rule
- 6.5 Sketching a Story about a Boy and a Bike
- 6.6 Comparing Volumes
- 6.7 Shadows
- 6.8 Drawing cross Sections
- 6.9 Finding Volume with Cubes, Can You Find Volume?
- 6.18 Playing with Cones

Provide access as needed throughout the unit:

	dependent variable radius	
Acc7.6.4	prediction	
Acc7.6.6	volume cube	
Acc7.6.7	functional relationship linear function mathematical model	function prediction
Acc7.6.8	cross section base (of a prism or pyramid) vertex (of a pyramid) face	prism pyramid perpendicular
Acc7.6.9		volume cross section base (of a prism or pyramid)
Acc7.6.11	cylinder three-dimensional base (of a cylinder or cone) approximation for π	radius
Acc7.6.12	dimension	base (of a cylinder or cone) cylinder
Acc7.6.13		surface area face
Acc7.6.16		cone
Acc7.6.19	hemisphere	
Acc7.6.20		sphere
Acc7.6.21	spherical	
Acc7.6.22	approximate range	

- Colored pencils
- Compasses
- Fruits or vegetables
- Geometry toolkits
- Knife
- Math Community Chart
- Paint
- Rulers marked with centimeters
- Snap cubes
- Spherical objects
- Straightedges
- Tools for creating a visual display

Lesson	Materials to Gather	Materials to Copy
1		Guess My Rule Cards (1 copy for every 4 students): Activity 2
5	Tools for creating a visual display: Activity 3	
6	Math Community Chart: Lesson	
8	• Fruits or vegetables: Activity 2 • Knife: Activity 2 • Paint: Activity 2	Cross Sections Cards (1 copy for every 3 students): Activity 3
9	• Snap cubes: Activity 2 • Rulers marked with centimeters: Activity 3	• Finding Volume with Cubes Handout (1 copy for every 6 students): Activity 2 • Can You Find the Volume Cutouts (1 copy for every 18 students): Activity 3
11	Colored pencils: Activity 3	
13		Multifaceted Cutouts (1 copy for every 30 students): Activity 1, Activity 2
14		Surface Area or Volume Cards (1 copy for every 2 students): Activity 4
19	Spherical objects: Activity 3	
21	• Tools for creating a visual display: Lesson • Math Community Chart: Activity 3	Unknown Dimensions Cards (1 copy for every 4 students): Activity 3
22	• Compasses: Activity 2 • Geometry toolkits: Activity 2, Activity 3 • Rulers marked with centimeters: Activity 2, Activity 3	
23	Straightedges: Activity 3	

Opportunities for Interdisciplinary Connections:

Physical Science: Students collect data to determine how the height of water is a function of volume when filling graduated cylinders

Anticipated misconceptions:

Dividing by Zero: Students may struggle to understand why a function has "no output" when a rule requires division by zero

See teacher's guide for specific misconceptions aligned to each

	lesson.
Connections to Prior Units:	**Connections to Future Units:**
Essential prior concepts to engage with this unit: • Variables representing quantities (Grade 6–7 Units 3–5) • Relationships between quantities (Grade 6, Unit 5 preview; Grade 7 Unit 5) • Tables, graphs, and equations as representations (Grade 6–7 Unit 5) • Measuring lengths with tools (Grades K–5) • Perimeter and area formulas (Grades 3–5) • Area of rectangles, triangles (Grades 3–6) • Recognizing 2D and 3D shapes (Grades K–5) • Understanding nets and surface area (Grade 6, Accelerated 6) • Volume of rectangular prisms (Grade 6, Accelerated 6) Understanding cylinders, cones, circles (Grade 7) • Multiplying with fractions and decimals (Grades 5–6, Accelerated 6) • Visualizing how 3D figures are built from 2D cross-sections (Grade 7 Unit 1, informally in earlier grades) **Relevant Unit(s)/Lesson(s) to Review:** • Start with the volume of rectangular prisms, this is foundational. Then review area formulas for circles since cylinder and cone volumes depend on this. • If students lack fluency with fraction and decimal operations, spend time on this before the unit; otherwise, arithmetic errors will mask conceptual understanding. • Visual 3D geometry work (naming shapes, recognizing cylinders vs. cones vs. spheres) should also be automatic.	Prepares students to study nonlinear relationships and how volume changes as dimensions increase

Differentiation through *Universal Design for Learning*

Engagement:
- Differentiate by having students complete only two lines of a table at a time before assessing comprehension (Lesson 16, Activity 2 Launch)
  LT6: Calculate the value of one dimension of a cone

Representation:
- Use color coding to highlight connections between temperatures for different cities that occurred at the same time (Lesson 6, Activity 1 Launch)
  LT3: Interpret multiple representations of functions

Action & Expression:
- Invite students to verbalize their strategy for completing a table quietly to themselves or a partner (Lesson 23, Activity 1 Launch)
  LT1: Comprehend the structure of a function

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:
- MLR2: Collect and Display (Lessons 2, 6, 8, 13)
- MLR1: Stronger and Clearer Each Time (Lessons 2, 9, 12, 17, 23)
- MLR7: Compare and Connect (Lessons 1, 5, 10, 13, 14, 20)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Movements of figures (Lessons 1 and 2).
- Observations about transforming parallel lines (Lesson 8).
- Transformations using corresponding points, line segments, and angles (Lesson 9).
- Observations about angle measurements (Lesson 14).
- Transformations found in tessellations and in designs with rotational symmetry (Lesson 18).

Generalize

- About categories for movement (Lesson 2).
- About rotating line segments 180° (Lesson 7).
- About the relationship between vertical angles (Lesson 8).
- About transformations and congruence (Lesson 11).
- About corresponding segments and length (Lesson 11).
- About alternate interior angles (Lesson 12).
- About the sum of angles in a triangle (Lesson 14).
- About categories for unique triangles (Lesson 16).

Justify

- Whether or not rigid transformations could produce an image (Lesson 6).
- Whether or not shapes are congruent (Lesson 10).
- Whether or not polygons are congruent (Lesson 11).
- Whether or not triangles can be created from given angle measurements (Lesson 13).
- Whether or not measurements determine unique triangles (Lesson 17).

Sentence Frames and Stems

Section A

- The rule between the input and the output of the function is _____ because ...
- If the input of the function is _____, then the output must be _____ because the rule is ...
- The equation _____ represents a function because ...
- The output of the function when the input is _____ is _____ because ...
- To draw the graph of a function that represents _____, first I _____, then I ...
- The graph represents a function because ...
- In the piecewise function, the rate of change from _____ to _____ is _____.
- The rate of change from _____ to _____ means _____.
- From _____ to _____ on the graph, _____ increases/decreases, which means ...
- The linear equation _____ can model the function because ...

Section B

- The shape of the cross section is _____ because...
- The base area of the prism is _____ and the height is _____, so the volume of the prism is _____. I calculated it by ...
- Figure _____ is/is not a prism because ...
- The surface area of the prism is _____ . I calculated it by ...
- In this situation, I need to find _____ (e.g. volume, base, surface area) because ...
- I know the volume of the cylinder/cone is _____, and the radius of the base is _____, so the height must be _____ because ...
- The volume of the cylinder is _____ because ...
- The volume of the cone is _____ because ...
- As the radius of the base increases by _____, the volume of the cylinder/cone increases by _____ because ...

Section C

- I know the volume of the cylinder/cone is _____, and the radius of the base is _____, so the height must be _____ because ...
- The volume of the cylinder is _____ because ...
- The volume of the cone is _____ because ...
- As the radius of the base increases by _____, the volume of the cylinder/cone increases by _____ because ...
- When the radius of the _____ is changed by _____, the volume ...
- The volume of the hemisphere is _____ because ...
- The volume of the sphere is _____ because ...

Section D

- The area of the triangle is _____.
- I chose this triangle because ...
- The volume/surface area of my prism is _____.

- The volume of a sphere with radius _____ is _____ the volume of a sphere with radius _____ because …
- The _____ can hold the highest volume of water because …

Unit Outline

In this unit, students are formally introduced to the concept of a function as a relationship between "inputs" and "outputs" in which each allowable input determines exactly one output. Due to the ordering of units in IM 6–8 Math Accelerated v.360, students may have been exposed informally to function terminology earlier in this course.

r	V
0	0
2	$\frac{32}{3}\pi$
6	288π
r	$\frac{4}{3}\pi r^3$

First, students work with relationships that are familiar from previous grades or units (perimeter formulas, proportional relationships, linear relationships), expressing them as functions. They study the different ways functions can be represented, making connections between the representations and interpreting what they mean in context. The use of function notation is left for a future course.

Next, students analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism and solve problems involving area, surface area, and volume. Students should have access to their geometry toolkits so that they have an opportunity to select and use appropriate tools strategically.

Students build on their knowledge of the formula for the volume of a right rectangular prism, learning formulas for volumes of cylinders, cones, and spheres. Students express functional relationships described by these formulas as equations, focusing on situations involving proportional relationships. They use these relationships to reason about how the volume of a figure changes as one of its dimensions changes, transforming algebraic expressions to get the information they need. In future courses, students will continue this thinking as they study nonlinear relationships and question how, for example, the volume of a sphere changes as the radius increases.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Representing and Interpreting Functions (Lessons 1-7)	**Learning Target #1:** Comprehend the structure of a function as having one and only one output for each allowable input. **Learning Target #2** Draw the graph of a function that represents a context, and explain which quantity is a function of which. **Learning Target #3** Interpret multiple representations of functions, including graphs, tables, and equations, and explain how to find information in each type of representation.	**Lesson 1 Inputs and Outputs** • I can write rules when I know input-output pairs. • I know how an input-output diagram represents a rule. **Lesson 2 Introduction to Functions** • I know that a function is a rule with exactly one output for each allowable input. • I know that if a rule has exactly one output for each allowable inpu then the output depends on the input. **Lesson 3 Equations for Functions** • I can find the output of a function when I know the input. • I can name the independent and dependent variables for a given function and represent the function with an equation. **Lesson 4 Graphs of Functions** • I can explain the story told by the graph of a function. • I can use a graph of a function to find the output for a given input and to find the input(s) for a given output. **Lesson 5 Even More Graphs of Functions** • I can draw the graph of a function that represents a real-world situation. **Lesson 6 Connecting Representations of Functions** • I can compare inputs and outputs of functions that are represent in different ways. **Lesson 7 Linear Functions and Models** • I can decide when a linear function is a good model for data and when it is not. • I can explain in my own words how the graph of a linear function relates to its rate of change and initial value. • I can use data points to model a linear function.
Checkpoint A	***Responding to Student Thinking*** • **Problem 1 & 2:** More Chances: Students will have more opportunities to develop this understanding later lessons. There is no need to slow down or add additional work to review this concept at this tim • **Problem 3:** Points to Emphasize: If most students struggle with drawing a graph of a function that represents a context, focus on this connection as opportunities arise. For example, in the practice problem referenced here, invite students to share their stories for each graph with a partner. ○ Accelerated 7, Unit 6, Lesson 9, Practice Problem 5	
Section B	**Learning Target #4**	**Lesson 8 Slicing Solids**

Prisms and Cylinders (Lessons 8-14)	Calculate the surface area and volume of a prism. **Learning Target #5** Calculate the volume of a cylinder.	• I can explain that when a three-dimensional figure is sliced it crea a face that is two dimensional. • I can picture different cross-sections of prisms and pyramids **Lesson 9 Volume of Right Prisms** • I can explain why the volume of a prism can be found by multiplyi the area of the base by the height of the prism. **Lesson 10 Decomposing Bases for Area** • I can calculate the volume of a prism with a complicated base by decomposing the base into quadrilaterals or triangles. **Lesson 11 The Volume of a Cylinder** • I can find the volume of a cylinder in mathematical and real-world situations. • I know the formula for the volume of a cylinder. **Lesson 12 Finding Cylinder Dimensions** • I can find missing information about a cylinder if I know its volume and some other information. **Lesson 13 Surface Area of Right Prisms** • I can decide whether I need to find the surface area or the volume when solving a problem about a real-world situation. • I can picture the net of a prism to help me calculate its surface are **Lesson 14 Applying Volume and Surface Area** • I can solve problems involving the volume and surface area of children's play structures.
Checkpoint B	***Responding to Student Thinking*** • **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of finding surfa area and volume of prisms. If most students struggle, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work. ○ Accelerated 7, Unit 6, Lesson 13 Surface Area of Right Prisms • **Problem 2:** Points to Emphasize: If most students struggle with using the formula for volume of a cylind revisit this skill throughout the next section. For example, during the Activity Synthesis of the activity referred to here, invite 1–3 students to share how they calculated the volume of the cylinder. ○ Accelerated 7, Unit 6, Lesson 19, Activity 2 Estimating Hemispheres	
Section C Cones and Spheres (Lessons 15-21)	**Learning Target #6** Calculate the value of one dimension of a cone, and explain the reasoning. **Learning Target #7** Calculate the volume of a cone or sphere. **Learning Target #8** Solve problems involving cones, cylinders, and spheres.	**Lesson 15 The Volume of a Cone** • I can find the volume of a cone in mathematical and real-world situations. • I know the formula for the volume of a cone. **Lesson 16 Finding Cone Dimensions** • I can find missing information about a cone if I know its volume ar some other information. **Lesson 17 Scaling One Dimension** • I can create a graph of the relationship between volume and heig for all cylinders and cones with a fixed radius. • I can explain in my own words why changing the height by a scale factor changes the volume by the same scale factor. **Lesson 18 Scaling Two Dimensions** • I can create a graph representing the relationship between volum and radius for all cones (or cylinders) with a fixed height. • I can explain in my own words why changing the radius by a scale factor changes the volume by the scale factor squared. **Lesson 19 Estimating a Hemisphere** • I can estimate the volume of a hemisphere by calculating the volu of a shape I know is larger and the volume of a shape I know is smaller. **Lesson 20 The Volume of a Sphere** • **I can find the volume of a sphere when I know the radius.** **Lesson 21 Cylinders, Cones, and Spheres** • I can find the radius of a sphere if I know its volume. • I can solve mathematical and real-world problems about the volu of cylinders, cones, and spheres.
Checkpoint C	***Responding to Student Thinking*** • **Problem 1:** Points to Emphasize: If most students struggle with calculating the radius of the cone, focus this type of solving as opportunities arise. For example, in the Warm-up referred to here, invite students calculate the radius of the cone if the volume was 9π or 72π (r=3 or r=6).	

	○ Accelerated 7, Unit 6, Lesson 19, Warm-up Notice and Wonder: Two Shapes ● **Problem 2:** Press Pause: If most students struggle with using the formula for volume, make time to do some or all of this optional lesson: ○ Accelerated 7, Unit 6, Lesson 23 Volume as a Function of ... ● **Problem 3:** Press Pause: If most students struggle with using formulas to order the four shapes by volu make time to do some or all of the optional lesson referred to here: ○ Accelerated 7, Unit 6, Lesson 23 Volume as a Function of ...	
Section D Let's Put it To Work (Lesson 19)	**No new learning targets**	**Lesson 22 Building Prisms** ● I can build a triangular prism from scratch. **Lesson 23 Volume as a Function of . . .** ● I can compare functions about volume represented in different ways.
End of Unit Assessment		