Course Title:	Content Area:	Grade Level:	Credit (if applicable)
Algebra 2 ACA/ACC	Math	Gr. 10/11	1.0

Course Description:

Algebra 2 extends students' understanding of functions, equations, and mathematical modeling through an in-depth exploration of polynomial, rational, exponential, and trigonometric functions while developing sophisticated problem-solving skills for real-world applications. Students progress through eight units covering sequences and functions, polynomial analysis with factors and zeros, rational functions and optimization, complex numbers and rational exponents, exponential and logarithmic relationships, function transformations, trigonometric modeling, and statistical inference. The course emphasizes multiple representations of mathematical relationships, logical reasoning, and mathematical communication while integrating technology for exploration and problem-solving. Students develop both procedural fluency and conceptual understanding through formative assessments, performance tasks, and mathematical discourse, with real-world connections spanning STEM applications, financial literacy, and data-driven decision making. By course completion, students demonstrate mastery of advanced algebraic concepts, confidence in tackling unfamiliar mathematical problems, and understanding of mathematics as an essential tool for understanding and improving the world around them.

Aligned Core Resources:

Teacher Materials
https://accessim.org

- Key Structures in this Course
  - Developing a Math Community
  - Student Journal Prompts
  - Mathematical Modeling Prompts
  - Are You Ready For More?
  - Principles of IM Curriculum Design
- Algebra 2 Scope and Sequence
  - Narrative
  - Unit 1: Sequences and Functions
  - Unit 2: Polynomial Functions
  - Unit 3: Rational Functions and Equations
  - Unit 4: Complex Numbers and Rational Exponents
  - Unit 5: Exponential Functions and Equations
  - Unit 6: Transformations of Functions
  - Unit 7: Trigonometric Functions
  - Unit 8: Statistical Inferences
  - Pacing Guide
  - Dependency Diagram

CT Core Standards

Imagine Learning iM Resources (BPS teacher login through ClassLink)

Connection to the *BPS Vision of the Graduate*

Common Core State Standards: Math Practice (MP) Standards
MP 1: Make sense of problems and persevere in solving them.
MP 2: Reason abstractly and quantitatively.
MP 3: Construct viable arguments and critique the reasoning of others.
MP 4: Model with mathematics.
MP 5: Use appropriate tools strategically.
MP 6: Attend to precision.
MP 7: Look for and make use of structure.
MP 8: Look for and express regularity in repeated reasoning.

Lessons that Showcase Math Practice Standards

	Unit 1 iM Alg 2 Unit 1	Unit 2 iM Alg 1 Unit 7	Unit 3 iM Alg 1 Unit 8	Unit 4 iM Alg 2 Unit 2	Unit 5 iM Alg 2 Unit 3	Unit 6 iM Alg 2 Unit 4
MP 1	5, 7, 10, 11	4, 14, 17	1-3, 9, 10, 21, 22, 24	14, 15	1, 6, 9–11	7, 8, 10, 15
MP 2	9, 10	1, 3, 6, 7, 12, 14	1, 2, 17, 21	1	3–6	8
MP 3	6	2, 3, 12, 15-17	5, 6, 13, 18, 19, 21	4, 5, 7, 9, 13	2	1, 6
MP 4	8, 9	7, 17	1			3, 8, 10, 15
MP 5	4–6, 8, 10	1, 4, 6, 12	3, 5, 17, 21	3, 9	2, 4–6, 11	6, 15
MP 6	1–3, 7, 8	7, 9, 11, 14, 15	9, 10, 13, 15, 22	1, 3, 6, 8, 9, 14	7	1, 2, 5, 7, 9, 11, 14
MP 7	1, 9, 11	1-5, 8-13, 15-17	3-13, 16, 19, 21, 23	2, 3, 5, 7, 9–11, 13	2, 3, 5, 7, 8, 10	4-7, 12-14
MP 8	1, 2, 5, 8	2, 5-7, 9-13, 15	4, 7, 8, 11, 14, 20, 21	8, 12, 15	1, 4, 8	2, 4, 10-12

Bristol Public Schools Vision of the Graduate

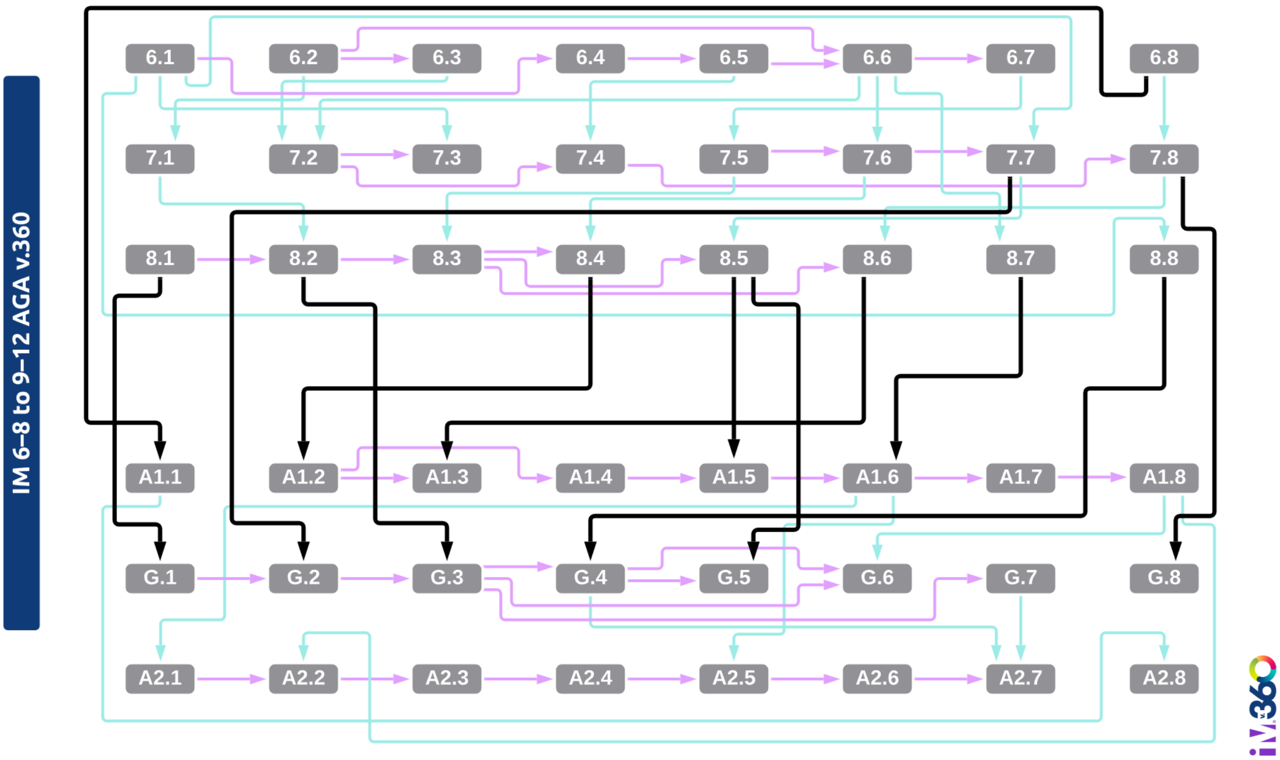
Problem Solving
- iM's focus on real-world modeling and problem-solving strategies
- Multiple solution pathways are encouraged and explored
- Students develop perseverance through challenging tasks

Critical Thinking
- Students analyze mathematical relationships and justify their reasoning
- Regular opportunities to critique others' reasoning
- Emphasis on understanding "why" not just "how"

Communication and Collaboration
- Structured mathematical discourse is built into lessons
- Students explain their thinking both verbally and in writing
- Many activities involve partner and group work

Link to *Completed Equity Audit*: Equity Curriculum Review Audit: 6-12 Illustrative Math (2026)

Additional Course Information: *Knowledge/Skill Dependent courses/prerequisites*



Standard Matrix

Standard	Unit 1 iM Alg 2 Unit 1	Unit 2 iM Alg 1 Unit 7	Unit 3 iM Alg 1 Unit 8	Unit 4 iM Alg 2 Unit 2	Unit 5 iM Alg 2 Unit 3	Unit 6 iM Alg 2 Unit 4
HSS-ID.A	X					
HSS-ID.A.1	X					
HSS-ID.A.2	X					
HSS-ID.A.3	X					
HSA-CED.A.2		X		X		
HSA-REI.D.10		X	X	X		X
HSA-SSE.A		X	X			
HSA-SSE.A.1		X	X	X		
HSA-SSE.B.3		X	X			
HSF-BF.A.1		X				
HSF.BF.A.1.a		X				
HSF-BF.B.3		X				
HSF-IF.A.2		X	X			
HSF-IF.B.4		X	X			
HSF-IF.B.6		X				
HSF-IF.C		X	X			
HSF-IF.C.7		X	X			
HSF-IF.C.7.a		X	X			
HSF-IF.C.8		X				

HSF-IF.C.8.a		X	X			
HSF-IF.C.9		X	X			
HSF-LE.A		X				
HSF-LE.A.1		X				
HSF-LE.A.2		X	X			
HSA-CED.A.1			X			X
HSA-CED.A.3			X	X		X
HSA-REI.A			X	X		
HSA-REI.A.1			X	X		
HSA-REI.B.3			X	X		X
HSA-REI.B.4			X			
HSA-REI.B.4.a			X			
HSA-REI.B.4.b			X			
HSA-REI.C.7			X			
HSA-REI.D			X			
HSA-SSE.A.2			X			
HSA-SSE.B.3.a			X			
HSA-SSE.B.3.b			X			
HSF-IF.A.1			X			
HSF-IF.B.5			X			
HSN-RN.B			X			
HSN-RN.B.3			X			
HSA-CED.A.4				X		
HSA-REI.C				X		
HSA-REI.C.5				X		
HSF-BF.A.1.b				X		
HSN-Q.A.3					X	
HSS-ID.B.5					X	
HSS-ID.B.6					X	
HSS-ID.B.6.a					X	
HSS-ID.B.6.b					X	
HSS-ID.B.6.c					X	
HSS-ID.C.7					X	
HSS-ID.C.8					X	
HSS-ID.C.9					X	
HSA-REI.D.12						X

Unit Links
Algebra 2 ACA/ACC Unit 1 (iM A2 U1): Sequences and Functions Unit 2 (iM A1 U7): Introduction to Quadratic Functions Unit 3 (iM A1 U8): Quadratic Equations Unit 4 (iM A2 U2): Polynomials Unit 5 (iM A2 U3): Rational Functions and Equations Unit 6 (iM A2 U4): Complex Numbers and Rational Exponents

Unit Title	
Unit 1 (iM A2 U1): Sequences and Functions	
Relevant Standards	

Lesson	Standards	Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 5	HSF-BF.A.1.a HSF-BF.A.2 HSF-IF.A.3 HSF-LE.A.2	Lesson 9	HSF-BF.A.2 HSF-IF.A.3 HSF-IF.B.5 HSF-LE.A.2
Lesson 2	HSF-BF.A.1.a	Lesson 6	HSF-BF.A.2 HSF-IF.C HSF-LE.A.2	Lesson 10	HSF-BF.A.2 HSF-LE.A.2
Lesson 3	HSF-IF.C	Lesson 7	HSF-BF.A.2 HSF-IF.A.3 HSF-LE.A.2	Lesson 11	HSF-BF.A.1.a HSF-BF.A.2
Lesson 4	HSF-IF.C	Lesson 8	HSF-BF.A.2 HSF-LE.A.2		

Essential Question(s)	**Enduring Understanding(s)**
• How do different representations (tables, graphs, recursive formulas, nth term formulas) help us understand sequences? • What are the advantages of recursive versus nth term definitions? • How can we move between different representations of the same sequence? • How can we determine if a sequence is arithmetic, geometric, or neither? • What real-world situations are best modeled by sequences rather than continuous functions?	• Sequences follow predictable patterns that can be identified, extended, and represented mathematically. • The growth characteristics of sequences can be quantified and used to make predictions. • Mathematical representations of sequences are interconnected and serve different purposes for analysis and communication. • Real-world contexts impose meaningful constraints on mathematical models. • Sequences provide powerful tools for modeling growth, decay, and change in authentic situations.
Demonstration of Learning	**Pacing for Unit**
CFA 1: Checkpoint A (after lesson 4) CFA 2: Checkpoint B (after lesson 7) Checkpoint C (after lesson 11) use as opportunity for feedback EoU: Assessment (after lesson 11)	16 Days (8 class periods) Lesson Modifications: (from adaptation pack) • Remove 1.4 - An optional lesson on spreadsheets • Remove 1.7 - info gap on reinforcing parts of the recursive function
Family Overview (link below)	**Integration of Technology:**
https://accessim.org/9-12-aga/algebra-2/unit-1?a=family	• Desmos Online Graphing Calculator
Unit-specific Vocabulary	**Aligned Unit Materials, Resources, and Technology (beyond core resources)**
sequence, term (of a sequence), geometric sequence, arithmetic sequence	• Desmos Online Graphing Calculator • Pear Assessment (Edulastic) • IL Classroom Adapted Resources • Interactive Tower of Hanoi puzzle • Paper cutting activity to visualize a sequence (paper and scissors)
Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions**
Science • Population growth models (geometric sequences for bacterial growth, predator-prey relationships): Cell division patterns and exponential reproduction, Fibonacci sequences in nature (plant growth patterns, shell spirals, flower petals), Ecosystem modeling with discrete time intervals • Population dynamics in ecological systems: Carbon	• Thinking sequences can have any real number input rather than just integers • Mixing up the term value with its position in the sequence • Looking at differences without checking if they're constant (arithmetic) or assuming any growing pattern is geometric

footprint reduction scenarios, Resource consumption modeling, Climate data analysis with discrete measurements Economics & Business • Compound interest calculations and investment growth: Depreciation models for assets, Market analysis using geometric and arithmetic progressions, Revenue and profit projections over time periods Art & Design • Fractal patterns and geometric art (Koch Snowflake connections): Architectural proportions and scaling, Visual pattern recognition and creation, Digital art algorithms and iterations	• Adding when they should multiply (geometric) or multiplying when they should add (arithmetic) • Expecting all sequences to be either arithmetic or geometric when many are neither • Writing $f(n-1)$ incorrectly or confusing it with $f(n)+1$ • Initial term confusion: Not clearly stating the starting value (e.g., $f(1)=3$) • Not being clear whether n starts at 0 or 1, leading to different but equivalent formulas • Not recognizing arithmetic sequences as linear functions with restricted domain • Not connecting geometric sequences to exponential functions • Difficulty understanding sequences that decrease (growth factor < 1) • Thinking a sequence is geometric when it involves both addition and multiplication • Difficulty moving from specific steps to general nth term formulas • Using inappropriate domains for real-world contexts • Mixing up terms like "growth factor," "common ratio," and "rate of change" See teacher's guide for specific misconceptions aligned to each lesson
Connections to Prior Units	**Connections to Future Units**
Relevant Unit(s) to review: Algebra 1: Units 5 and 6 • Alg 1 Unit 5 Lesson 2 Activity 1 • Alg 1 Unit 5 Lesson 4 • Alg 1 Unit 6 Lesson 3 Warm-up (pay particular attention to the review in the launch) Essential prior concepts to engage with this unit • Function notation. • Translating among table, graph, function, and description for both linear and exponential situations.	Connection to Unit 4 (iM A2 U2): Polynomial Functions • Building explicit formulas • Function representation skills • Domain understanding:

Differentiation through *Universal Design for Learning*

Engagement
- Connect sequences to real-world contexts and familiar patterns students encounter daily
- Offer multiple entry points and representations to accommodate different learning preferences
- Scaffold learning progression systematically from concrete examples to abstract notation
- Build on prior knowledge of linear and exponential functions while celebrating incremental progress
- Make learning goals transparent and provide opportunities for reflection on different representations Encourage metacognitive strategies and mathematical discourse about sequence patterns and formulas.

Representation
- Present sequences through multiple sensory modalities (visual graphs, numerical tables, written patterns)
- Use clear visual design in coordinate plane representations and ensure accessibility of mathematical notation
- Support vocabulary development for key terms (sequence, term, recursive, explicit, arithmetic, geometric)
- Provide multiple representations of mathematical notation and connect symbolic expressions to verbal descriptions
- Activate prior knowledge by explicitly connecting sequences to familiar linear and exponential functions
- Guide pattern recognition through scaffolded examples and highlight critical features that distinguish arithmetic from geometric sequences

Action & Expression
- Provide multiple ways for students to demonstrate understanding (verbal explanations, written formulas, graphical representations, or digital tools)
- Support planning and strategy development by teaching systematic approaches to identify sequence types and choose appropriate representations
- Offer various tools and assistive technologies for mathematical expression including graphing calculators, spreadsheets, and function notation templates

- Scaffold executive functions by providing organizational frameworks for moving between recursive and explicit formulas
- Create opportunities for students to express mathematical thinking through multiple modalities and support fluency development in function notation
- Guide students in monitoring their progress as they transition from informal pattern description to formal mathematical modeling

Supporting Multilingual/English Learners

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problem
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

MLR1: Stronger and Clearer Each Time
Used in Lesson 1 (Tower of Hanoi) where students refine their mathematical language through multiple drafts when describing sequences
MLR2: Collect and Display
Used in Lesson 3 (Three Sequences) to capture and organize student language about different types of sequences
MLR5: Co-Craft Questions
Used in Lesson 2 (Paper Slicing) where students generate mathematical questions about geometric sequences
MLR6: Three Reads
Used in Lesson 8 (Sierpinski Triangle) to help students analyze complex mathematical text about sequence definitions
MLR7: Compare and Connect
Used in Lesson 5 (Let's Define Some Sequences) where students compare different recursive definitions and representations
MLR8: Discussion Supports
Used in multiple lessons (Lessons 6, 11) to support rich mathematical discussions about sequences and their representations

Sentence Frames and Stems

Section A

- The next term(s) in the sequence _____ (list the given terms) is/are _____ because ...
- I know the sequence _____ (list the given terms) is arithmetic/geometric because ...
- The pattern is growing by _____ which represents a/an _____ sequence because ...
- The rate of change in the arithmetic sequence is _____ because ...
- The growth factor in the geometric sequence is _____ because ...

Section B

- The recursive definition f(1) = _____, f(n) = _____ ["f of 1" is _____, "f of n" is _____] describes this sequence because ...
- To create a graph of the sequence _____ (list the given terms), first I _____, then I ...
- The table shows the _____ terms which are _____ by _____. This means the sequence is _____ because ...
- Given the recursive definition f(1) _____, f(n) _____ ["f of 1" is _____, "f of n" is _____], the first _____ terms of the _____ sequence are ...

Section C

- The formula _____ defines the sequence _____ (list the given terms) because ...
- The function _____ can be used to find the nth term in the sequence _____ (list the given terms) because ...
- The function _____ can be used to represent this situation because ...
- A reasonable domain for this situation is _____ because ...

Related CELP standards and Learning Targets:

	Emerging	Expanding	Bridging
LT1	Find the next two terms by counting or simple addition/multiplication, using short phrases, single words, or drawings to communicate the rule and answer. Focus is on sequences with small whole numbers.	Find a missing term in the middle of a sequence, calculating with integers and simple decimals. Uses short, simple sentences with general academic terms (e.g., "I added the number four").	Find any missing term (nth term) using the explicit formula. Articulates and justifies the process using complex sentences and content-specific language (e.g., "I used the explicit formula $a_n=a_1+(n-1)d$").
LT2	Identify the growth factor/rate when given adjacent terms that are small whole numbers. Verbalizes or writes the answer as a number or simple	Calculate the rate of change (d) or growth factor (r) given a table or two terms, including sequences with negative numbers. Explains the	Derive and justify the rate of change or growth factor from non-adjacent terms or a complex verbal situation. Uses precise technical vocabulary ("common

	phrase (e.g., "plus 5" or "times 2").	calculation using connected, but simple, sentences.	difference," "common ratio") and defends the solution in a coherent paragraph.
LT3	Complete a pre-made table or graph by filling in a few steps of a sequence given the initial term and the rule, using a template or heavy scaffolding.	Create a complete table and accurately sketch a graph OR write a basic recursive rule for a sequence, using short sentences to explain the pattern.	Independently create an accurate table, graph, AND a recursive rule for a sequence based on a complex context, using well-structured language to define the relationship between consecutive terms.
LT4	Insert numerical values into a provided explicit equation template or sentence frame to represent a simple situation. Focus on correct substitution (e.g., f(n)=[start]+n · [rate]).	Write the explicit equation for an arithmetic or geometric sequence from a given table or scenario, using appropriate function notation.	Develop and write both the explicit and recursive equations for a sequence representing a complex real-world situation, and clearly articulate in writing the meaning of each variable and why the equation is appropriate.
LT5	State the starting point (n=1 or n=0) and correctly identify that the domain uses whole numbers, possibly by pointing or circling options.	Explain, using simple sentences, why the domain must be integers (i.e., discrete data) based on the context (e.g., "Because you only have whole stairs, not half a stair").	Analyze a complex scenario to determine, state, and mathematically justify the full domain and range restrictions (start, end, and integer-only) using precise set notation.

Unit Description

This unit provides students an opportunity to revisit functions by way of sequences. Through many concrete examples, students will see arithmetic and geometric sequences as linear and exponential functions restricted to a domain that is a subset of the integers. This unit reinforces understanding of functions by using multiple representations (including graphs, tables, and expressions).

Students begin with an invitation to describe sequences with informal language and learn to identify if a sequence is geometric, arithmetic, or neither. Students write out the terms of sequences arising from mathematical situations, in addition to interpreting and creating tables and graphs about the given relationship.

Students learn that sequences are a type of function in which the input variable is the position and the output variable is the term at that position. They learn to interpret sequences and then use function notation to write their own recursive definitions of the sequences. Students make connections between the sequences and different representations of functions.

Students then build on their prior knowledge of linear and exponential functions to develop explicit formulas for arithmetic and geometric sequences and to model several situations represented in different ways.

In the last section of the unit, students use sequences to model several situations represented in different ways. This work is meant to touch on some practices that must be attended to while modeling, such as choosing a good model and identifying an appropriate domain. Students also recognize that a sequence is an appropriate type of function to use as a model for these situations since the domain of each situation is a subset of the integers. Finally, students encounter some situations in which it makes sense to compute the sum of a finite sequence. This thinking connects to later work in which students will determine the formula for the sum of the first terms of a geometric sequence.

Lesson Sequence	Learning Target	Success Criteria/Assessment
Section A Sequences (Lessons 1-4)	**Learning Target 1** Determine missing terms in arithmetic and geometric sequences. **Learning Target 2** Determine the growth factor of a geometric sequence and the rate of change of an arithmetic sequence.	**Lesson 1 A Towering Sequence** • I can give an example of a sequence. **Lesson 2 Introducing Geometric Sequences** • I can find missing terms in a geometric sequence. **Lesson 3 Different Types of Sequences** • I can explain what it means for a sequence to be arithmetic or geometric. Checkpoint A Problems 1 and 2 ACC **Lesson 4 Using Technology to Work with Sequences**

<table>
<tr><td></td><td></td><td>I can use a spreadsheet to create many terms of a sequence.
I can use technology to graph a sequence.</td></tr>
<tr><td>Checkpoint A</td><td colspan="2">Responding to Student Thinking
Problem 1 & 2 (remove 1c)
More Chances - Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.</td></tr>
<tr><td>Section B
Representing Sequences
(Lessons 5-7)</td><td>Learning Target 3
Create a table, graph, or recursive definition of a sequence from given information.</td><td>Lesson 5 Sequences Are Functions
I can use function notation to define arithmetic and geometric sequences recursively.
Checkpoint B Problem 1
End of Unit Problems 1 and 4
Lesson 6 Representing Sequences
I can represent a sequence in different ways.
ACC Lesson 7 Representing More Sequences
I can ask questions to get the information needed to represent a sequence in different ways.</td></tr>
<tr><td>Checkpoint B</td><td colspan="2">Responding to Student Thinking
Points to Emphasize - If most students struggle to write a recursive definition, spend more time asking students to share their strategies for writing a recursive definition in the Activity Synthesis of the activity referred to here.</td></tr>
<tr><td>Section C
What's the Equation?
(Lessons 8-11)</td><td>Learning Target 4
Create equations for sequences representing situations.

Learning Target 5
Identify restrictions on the domain of a sequence based on the context.</td><td>Lesson 8 The nth Term
I can explain why different equations can represent the same sequence.
End of Unit Problems 2, 3, 6 and 7
Lesson 9 What's the Equation?
I can represent situations with sequences.
Checkpoint B Problem 1 (feedback opportunity)
End of Unit Problem 5
Lesson 10 Situations and Sequences Types
I can define a sequence using an equation.
Lesson 11 Adding Up
I can determine the sum of a sequence representing a situation.</td></tr>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
Press Pause - By this point in the unit, there should be some student mastery of writing equations for sequences representing situations. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work. For example, consider having students create their own patterns with black and white squares, similar to those in the activity referred to here, and exchange with a partner to write equations for the patterns.</td></tr>
</table>

END OF UNIT ASSESSMENT

General End of Unit Assessment Notes:

- Modify select all questions to mimic SAT-style "I-II-III Questions"
- #1 - Press enter after each part of the formula
- #2 - Make them all (n-1) and switch the start at and rate of change in B and D
- #3 - make some n and some n-1, but make the rate of change and start at correct on all, remove E and F, make SAT-like but only 1 correct answer
- #4 - do not include domain restrictions/index, tell them it's arithmetic, put blank in the table between 1 and 3 (optional)
- #5 - A strip that is ⅓ the orignal area is cut off, and the remaining area is recorded. Part c) Reword: What is a reasonable domain for this situation? Justify why you included some values of n and not others in your domain.
- #6 - Add a fill in the blank for rate of change before each definition (0.5 pt), definition as (1.5 pts)
- #7 - Rephrase and b: Use words to describe how to get from one term to the next, make everything with Sequence A together and B together. Add back in/keep Which is greater A(9) or B(9)

Unit Title:
Unit 2 (iM A1 U7): Introduction to Quadratic Functions Also see Algebra 1 ACA Unit 6/Alg+ Unit 8 (Algebra 1 is aiming to cover through Lesson 11)

Relevant Standards

Lesson	Standards	Lesson	Standards	Lesson	Standards
Lesson 1	HSF-BF.A.1.a	Lesson 7	HSF-BF.A.1.a HSF-IF.B.5 HSF-IF.C.7.a	Lesson 13	HSA-SSE.B.3 HSF-BF.B.3 HSF-IF.C.7 HSF-IF.C.7.a
Lesson 2	HSA-SSE.A.1 HSA-SSE.B.3 HSF-BF.A.1.a HSF-IF.A.3	Lesson 8	HSA-APR.A HSA-SSE.A HSA-SSE.A.2 HSA-SSE.B.3	Lesson 14	HSF-IF.A.2 HSF-IF.B.4 HSF-IF.C.7.a HSF-IF.C.8 HSF-IF.C.9
Lesson 3	HSA-SSE.A.1 HSF-BF.A.1.a HSF-IF.A.2	Lesson 9	HSA-APR.A HSA-SSE.A.2 HSA-SSE.B.3	Lesson 15	HSF-BF.B.3 HSF-IF.C HSF-IF.C.7.a
Lesson 4	HSF-BF.A.1.a HSF-IF.C HSF-LE.A.3	Lesson 10	HSA-SSE.B.3	Lesson 16	HSF-IF.C HSF-IF.C.7.a
Lesson 5	HSF-BF.A.1.a HSF-IF.A.2	Lesson 11	HSA-SSE.A HSF-IF.C.7.a	Lesson 17	HSF-BF.B.3 HSF-IF.C HSF-IF.C.7.a
Lesson 6	HSF-BF.A.1 HSF-BF.A.1.a HSF-IF.B.5 HSF-IF.C HSF-IF.C.7.a	Lesson 12	HSF-BF.B.3 HSF-IF.CHSF-IF.C.7 HSF-LE.A.2		

Essential Question(s):

- How do different forms of a function's equation help us understand its key features and behavior?
- What do the key features of a function—such as intercepts, maximums, minimums, and end behavior—reveal about its graph and real-world meaning?
- How do transformations, such as shifts, reflections, and stretches, affect the graph and equation of a function?
- How do we determine whether a function best models a real-world situation as linear, quadratic, or exponential?
- What are the different kinds of growth and how do they compare?
- How can we analyze and compare functions that are represented in different ways, such as equations, graphs, tables, and verbal descriptions?
- How can expressing the same equation in different forms reveal different properties of a quadratic function?

Enduring Understanding(s):

- Functions can be represented in multiple ways and each representation provides different insights into the function's behavior.
- Rewriting expressions through factoring, expanding, and completing the square reveals important characteristics of functions, such as zeros, intercepts, and vertex points.
- Transformations, including shifts, reflections, stretches, and compressions, help us understand how function graphs change and how different functions relate to one another.
- The structure of an algebraic expression can be analyzed and rewritten to make solving equations easier and to reveal key features of the function it represents.
- Exponential, linear, and quadratic functions model different types of real-world relationships, and understanding their differences allows us to choose the best function for a given situation.
- Comparing functions in different forms—such as equations, tables, graphs, and descriptions—helps us analyze relationships and make predictions.
- Projectile motion can be represented using a quadratic function.

Demonstration of Learning:	Pacing for Unit
Section A Checkpoint	21 Days (11 class periods)

<table>
<tr><td>Section B Checkpoint
Section C Checkpoint
Mid-Unit Assessment
Section D Checkpoint
End-of-Unit Assessment</td><td>Lesson Modifications: (from adaptation pack)
No lessons are recommended to remove or modify for additional lessons. If time is an issue, these modifications could be made to move students more quickly into working with quadratic equations in the next unit:
Combine Lessons 1 and 2. Emphasize Activities 2.2 and 2.3.
Combine Lessons 3 and 4. Emphasize Activity 4.2 and the idea that exponential functions always eventually grow more quickly than quadratic functions.
Remove Lesson 7. This focuses on quadratic functions in the context of price versus revenue.
Remove Lesson 13. This focuses on the effect of the linear term in quadratic functions and is beyond the scope of Algebra 1 standards.
Remove Lessons 15–17. These lessons introduce vertex form. While vertex form is important, the lessons in the first sections of Unit 7 are of higher priority for Algebra 1 students.</td></tr>
<tr><td>Family Overview (link below)</td><td>Integration of Technology:</td></tr>
<tr><td>https://accessim.org/9-12-aga/algebra-1/unit-7?a=family</td><td>Desmos Online Graphing Calculator</td></tr>
<tr><td>Unit-specific Vocabulary:</td><td>Aligned Unit Materials, Resources, and Technology (beyond core resources):</td></tr>
<tr><td>Quadratic expression, quadratic function, vertex, zero, factored form, standard form, vertex form</td><td>Desmos Online Graphing Calculator
Pear Assessment (Edulastic)
IL Classroom Adapted Resources</td></tr>
<tr><td>Opportunities for Interdisciplinary Connections:</td><td>Anticipated misconceptions:</td></tr>
<tr><td>Science
Quadratic functions describe projectile motion and acceleration in physics, helping students understand real-world applications of parabolic paths.
Transformations of functions relate to wave motion, sound frequencies, and light reflection in physics.
Economics & Business
Linear and quadratic models help analyze profit, revenue, and cost trends in business scenarios.
Understanding function transformations allows for predicting market growth and stock trends over time.
Computer Science & Engineering
Quadratic and exponential functions play a role in algorithm efficiency and machine learning models in computer science.
Graph transformations and function comparisons help analyze digital image processing, scaling, and animation effects in game development.</td><td>Many students misunderstand that transformations (shifts, reflections, stretches) apply to all points on a function, not just specific ones like the vertex.
Some students assume that exponential and quadratic functions both grow at a constant rate instead of recognizing that exponentials grow by multiplication while quadratics do not.
Students may incorrectly believe that the x-intercepts (solutions) are always the maximum or minimum of the function, rather than understanding how the vertex relates to symmetry and extrema.
Some students struggle to compare functions across different representations (graph, table, equation, verbal) and focus only on algebraic form.
See teacher's guide for specific misconceptions aligned to each lesson</td></tr>
<tr><td>Connections to Prior Units:</td><td>Connections to Future Units:</td></tr>
<tr><td>Prior to this unit, students have studied what it means for a relationship to be a function, used function notation, and investigated linear and exponential functions. In this unit, they look at some patterns that grow quadratically and contrast this growth with linear and exponential growth.

Essential prior concepts to engage with this unit: Write equivalent expressions by combining like terms and applying the distributive property.
If students need additional support with combining like terms or applying the distributive property, Grade 7 Unit 6 Lessons 18–22 address these topics.</td><td>Connections to Units in Future Courses

Transformations of Functions
Understanding shifts, reflections, stretches, and compressions of functions lays the foundation for geometric transformations in Geometry and function transformations in Algebra 2.
Circles, Higher-Degree Polynomials, and Rational Expressions
Skills like factoring and completing the square extend to deriving the equation of a circle and factoring and solving higher-degree polynomials in Algebra 2.</td></tr>
</table>

• The Algebra 1 Extra Supports materials, particularly Lessons 1 and 3, provide additional material for students who may need resources to support their understanding of area and perimeter.	

Differentiation through *Universal Design for Learning*

Engagement
- Connect rational exponents to familiar geometric contexts like area and volume calculations to make abstract concepts concrete
- Build on familiar integer exponent rules to justify and extend understanding to fractional exponents
- Address the conceptual challenge of imaginary numbers by introducing them as solutions to previously unsolvable equations
- Scaffold learning progression from square and cube roots to rational exponents to complex number operations
- Make learning goals transparent about extending the number system and solving all quadratic equations
- Encourage mathematical discourse about why extraneous solutions arise when squaring both sides of equations

Representation
- Present complex numbers through multiple sensory modalities using the imaginary axis, coordinate plane representations, and algebraic notation
- Ensure clear visual design for representing square roots, cube roots, and rational exponent notation
- Support vocabulary development for terminology (rational exponent, imaginary number, complex number, extraneous solution, principal root)
- Connect symbolic radical expressions to verbal descriptions and geometric interpretations of roots
- Activate prior knowledge by explicitly connecting rational exponents to familiar exponent rules and root operations
- Guide pattern recognition in complex number operations and highlight critical features like $i^2 = -1$

Action & Expression
- Provide multiple ways for students to demonstrate understanding through radical equations, complex arithmetic, or geometric applications
- Support planning strategies for choosing appropriate methods to solve quadratic equations including those with complex solutions
- Offer various tools including graphing technology, exponent rule organizers, and complex number operation templates
- Scaffold executive functions by providing organizational frameworks for connecting rational exponents to radical notation
- Create opportunities for students to express mathematical thinking about extending the number system beyond real numbers

Supporting Multilingual/English Learners

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

MLR1: Stronger and Clearer Each Time
Used in Lesson 9 (Standard Form and Factored Form) where students refine their explanations of different forms, and Lesson 16 (Graphing from the Vertex Form) where students improve their descriptions of graphing strategies

MLR2: Collect and Display
Used in Lesson 3 (Expanding Squares) to capture student language about quadratic patterns, Lesson 5 (Galileo and Gravity) for language about falling objects, and Lesson 11 (What Do We Need to Sketch a Graph?) to organize terminology about graph features

MLR5: Co-Craft Questions
Used in Lesson 5 (Falling from the Sky) where students generate mathematical questions about free-falling objects and quadratic motion

MLR6: Three Reads
Used in Lesson 17 (A Peanut Jumping over a Wall) to help students analyze complex contextual problems involving quadratic functions

MLR7: Compare and Connect
Used in Lesson 8 (Using Diagrams to Find Equivalent Quadratic Expressions) where students compare different strategies for expanding expressions and connect area diagrams to algebraic methods

MLR8: Discussion Supports
Used extensively throughout multiple lessons (Lessons 1, 2, 9, 15, 16) to support rich mathematical discussions about quadratic

patterns, different forms of expressions, and graph transformations

Sentence Frames and Stems

Section A

- This pattern represents a _____ relationship because ...
- Using the _____ I see that the pattern is growing by ...
- The expression _____ represents this situation because ...

Section B

- The expression _____ represents this situation because ...
- When the input is _____, the output for the function _____ is greater/less than the output for the function _____ because ...
- The height of the _____ after _____ is _____ because ...
- The function _____ represents this situation because ...

Section C

- If the factored form of a quadratic expression is _____, then the standard form is _____ because ...
- In this situation, the _____ (feature of the graph) represent(s) _____ because ...
- The equation _____ defines the function of the graph shown because ...

Section D

- When the equation is written in _____ form, I can easily identify the _____ (feature of the graph) which is _____ because ...
- In the equation _____, if the value _____ is changed to _____, the graph changes by ...
- To graph the function _____, first I _____, then I ...
- In this situation, the _____ (feature of the graph) represent(s) _____ because ...
- An EL can . . . construct grade appropriate oral and written claims and support them with reasoning and evidence.

Related *CELP standards* and Learning Targets:

	Emerging	Expanding	Bridging
LT1	I can identify quadratic expressions by looking for an x^2 term using a visual checklist and word bank.	I can explain that a relationship is quadratic when it contains a squared term (such as x^2) using sentence frames.	I can define and explain how a squared term (x^2) creates a non-linear quadratic relationship in equations, tables, and graphs.
LT2	I can sort visual patterns into linear, exponential, or quadratic groups using pattern cards and graphic organizers.	I can explain whether a visual pattern is linear, exponential, or quadratic by describing how it grows using sentence starters.	I can analyze visual patterns and construct a mathematical argument proving whether the growth rate is linear, exponential, or quadratic.
LT3	I can match points on a projectile graph (starting height, peak, landing point) to picture cards or simple labels.	I can describe what key parts of a quadratic graph mean in real-world contexts (e.g., maximum height, time in air) using sentence frames.	I can analyze quadratic expressions and graphs modeling physical phenomena to interpret key features (vertex, intercepts, domain) in context.
LT4	I can compare tables or graphs to point out where an exponential line passes above a quadratic curve using visual guides.	I can use tables, graphs, and calculations to show how an exponential function eventually becomes larger than a quadratic function using sentence starters.	I can demonstrate and justify numerically and graphically why exponential growth will always eventually exceed quadratic growth.
LT5	I can match the numbers in a factored form equation, like $(x - a)(x - b)$, to the x-intercepts on a graph using color-coding.	I can explain how to find the x-intercepts of a parabola from its factored form equation using step-by-step guides and sentence frames.	I can connect the zero product property to factored form quadratic equations to determine and explain the x-intercepts of a graph.
LT6	I can use an area model (box method) to multiply two binomials and write an equivalent expression with visual templates.	I can use the distributive property or area model to convert factored form into standard form ($ax^2 + bx + c$) using guided prompts.	I can algebraically convert quadratic expressions from factored form into standard form using the distributive property and prove their equivalence.
LT7	I can match parameter changes in an equation to visual movement of a parabola (shifts up, down, flips) using visual cards.	I can describe how changing numbers in standard, factored, or vertex form changes a graph's shape or position using sentence starters.	I can explain in detail how changing parameters a, b, c, h, k, and roots affects the graph's orientation, width, vertex, and intercepts across forms.
LT8	I can circle the vertex point (h, k) in an equation like $f(x) = a(x - h)^2 + k$ and point to the high or low point on a graph.	I can identify the maximum or minimum value of a quadratic function from its vertex form equation using structured sentence frames.	I can analyze a quadratic equation in vertex form $f(x) = a(x - h)^2 + k$ to determine its vertex, identify if it is a maximum or minimum, and justify why using the sign of a.

Unit Description

Prior to this unit, students have studied what it means for a relationship to be a function, used function notation, and investigated linear and exponential functions. In this unit, they look at some patterns that grow quadratically and contrast this growth with linear and exponential growth. They further observe that eventually these quadratic patterns grow more quickly than do linear patterns but more slowly than exponential patterns grow.

Students examine the important example of free-falling objects whose height over time can be modeled with quadratic functions. They use tables, graphs, and equations to describe the movement of these objects, eventually looking at the situation in which a projectile is launched upward. They interpret the meaning of each term in this context and work toward understanding how the coefficients influence the shape of the graph. Additional situations, such as revenue and area, are also introduced.

Next, students examine standard, factored, and vertex forms of quadratic functions. They recognize what information about the graph is easily obtained from each form and how the different values in each form influence the graph. In particular, they begin to generalize ideas of how horizontal and vertical translation, as well as vertical and horizontal stretching of graphs, relate to modifying the equation of a function.

Note on materials: Access to graphing technology is necessary for many activities. Examples of graphing technology are: a handheld graphing calculator, a computer with a graphing calculator application installed, and an internet-enabled device with access to a site like desmos.com/calculator or geogebra.org/graphing. For students using the digital version of these materials, a separate graphing calculator tool isn't necessary. Interactive applets are embedded throughout, and a graphing calculator tool is accessible in the student math tools.

Lesson Sequence	Learning Target	Success Criteria/Assessment
Section A: A Different Kind of Change Lessons 1-2	**Learning Target 1** I can comprehend that a "quadratic relationship" can be expressed with a squared term. **Learning Target 2** I can determine and explain whether a visual pattern represents a linear, exponential, or quadratic relationship.	**Lesson 1 A Different Kind of Change** • I can create drawings, tables, and graphs that represent the area of a garden. • I can recognize a situation represented by a graph that increases and then decreases. **Lesson 2 How Does it Change?** • I can describe how a pattern is growing. Checkpoint B Problem 1 Middle of Unit Problem 1 • I can tell whether a pattern is growing linearly, exponentially, or quadratically. Checkpoint A Middle of Unit Problem 1 • I know a quadratic expression has a squared term. Checkpoint A
Checkpoint A	*Responding to Student Thinking* **Problem 1:** More Chances: Students will have more opportunities to develop this understanding in later sections. There is no need to slow down or add additional work to review this concept at this time.	
Section B Quadratic Functions Lessons 3-7	**Learning Target 3** I can interpret quadratic functions that represent a physical phenomenon, given expressions and graphs. **Learning Target 4** I can use graphs, tables, and calculations to show that exponential functions eventually overtake quadratic functions.	**Lesson 3 Building Quadratic Functions from Geometric Patterns** • I can explain using graphs, tables, or calculations that exponential functions eventually grow faster than quadratic functions. Middle of Unit Problem 5 • I can recognize quadratic functions written in different ways. • I can use information from a pattern of shapes to write a quadratic function. Middle of Unit Problem 1

		Lesson 4 Comparing Quadratic and Exponential Functions • I can explain using graphs, tables, or calculations that exponential functions eventually grow faster than quadratic functions. Middle of Unit Problem 5 **Lesson 5 Quadratic Functions to Describe Situations (Part 1)** • I can explain the meaning of the terms in a quadratic expression that represents the height of a falling object. Checkpoint B Problem 2 Middle of Unit Problem 2 • I can use tables, graphs, and equations to represent the height of a falling object. **Lesson 6 Building Quadratic Functions to Describe Situations (Part 2)** • I can create quadratic functions and graphs that represent a situation. • I can relate the vertex of a graph and the zeros of a function to a situation. • I know that the domain of a function can depend on the situation it represents. Middle of Unit Problem 2 **ACC Lesson 7 Building Quadratic Functions to Describe Situations (Part 3)** • I can choose a domain that makes sense in a revenue situation. Middle of Unit Problem 7 • I can model revenue with quadratic functions and graphs. Middle of Unit Problem 4 *NOTE: Middle of Unit Problem 4* *Test question context is area but the Lesson is based on revenue, may want to preview revenue.* Middle of Unit Problem 7 • I can relate the vertex of a graph and the zeros of a function to a revenue situation.
Checkpoint B	*Responding to Student Thinking* **Problem 1:** Points to Emphasize: If students struggle to describe why exponential growth is eventually faster than quadratic growth, plan to revisit the idea throughout the rest of the unit. For example, as students explore graphs of quadratic functions in the lesson referred to here, use different functions of the form $y = ab^x$ to show that the exponential functions eventually grow faster. **Problem 2:** More Chances: Students will have more opportunities to develop this understanding in later sections. There is no need to slow down or add additional work to review this concept at this time.	
Section C Working with Quadratic Expressions Lessons 8-10	**Learning Target 5** I can coordinate a quadratic expression given in factored form and the intercepts of its graph. **Learning Target 6** I can use the distributive property to write equivalent quadratic expressions from factored into standard form.	**Lesson 8 Equivalent Quadratic Expressions** • I can rewrite quadratic expressions in different forms by using an area diagram or the distributive property. Checkpoint C Middle of Unit Problem 3 Middle of Unit Problem 6 **Lesson 9 Standard Form and Factored Form** • I can rewrite quadratic expressions given in factored form, in standard form, using either the distributive property or a diagram. Middle of Unit Problem 3

<table>
<tr><td></td><td></td><td>Middle of Unit Problem 6

- I know the difference between “factored form” and “standard form.”

Checkpoint C

Lesson 10 Graphs of Functions in Standard and Factored Forms

- I can explain the meaning of the intercepts on a graph of a quadratic function in terms of the situation it represents.

Middle of Unit Problem 7
End of Unit Problem 5
NOTE: End of Unit Problem 5 Clare’s function in factored form should be looked at. Question b. Give a different time value so that someone’s height is higher and they are NOT the same. Difficult to understand why a student made an error.
End of Unit Problem 7

- I know how the numbers in the factored form of a quadratic expression relate to the intercepts of its graph.

Middle of Unit Problem 7
End of Unit Problem 5
End of Unit Problem 7</td></tr>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
Problem 1: More Chances: Students will have more opportunities to develop this understanding in later sections. There is no need to slow down or add additional work to review this concept at this time.</td></tr>
<tr><td>Section D
Features of Graphs of Quadratic Functions

Lessons 11-17</td><td>Learning Target 7
I can explain how a graph is affected by changing parameters in quadratic expressions written in standard, factored, and vertex forms.

Learning Target 8
I can use an equation in vertex form to identify the maximum or minimum of a quadratic function.</td><td>Lesson 11 Graphing from the Factored Form

- I can graph a quadratic function given in factored form.

End of Unit Problem 4

- I know how to find the vertex and y-intercept of the graph of a quadratic function in factored form without graphing it first.

End of Unit Problem 4
End of Unit Problem 5

Lesson 12 Graphing the Standard Form (Part 1)

- I can explain how the a and c in $y = ax^2 + bx + c$ affect the graph of the equation.

Checkpoint D Problem 1
End of Unit Problem 3

- I understand how graphs, tables, and equations that represent the same quadratic function are related.

ACC Lesson 13 Graphing the Standard Form (Part 2)

- I can explain how the b in $y = ax^2 + bx + c$ affects the graph of the equation.
- I can match equations given in standard and factored form with their graph.

End of Unit Problem 1
End of Unit Problem 2

Lesson 14 Graphs That Represent Situations I can explain how a quadratic equation and its graph relate to a situation.

End of Unit Problem 5

ACC Lesson 15 Vertex Form</td></tr>
</table>

		• I can recognize the "vertex form" of a quadratic equation. • I can relate the numbers in the vertex form of a quadratic equation to its graph. End of Unit Problem 2 End of Unit Problem 6 **ACC Lesson 16 Graphing from the Vertex Form** • I can graph a quadratic function given in vertex form, showing a maximum or minimum and the y-intercept. • I know how to find a maximum or a minimum of a quadratic function given in vertex form without first graphing it. Checkpoint D Problem 2 End of Unit Problem 6 **ACC Lesson 17 Changing the Vertex** • I can describe how changing a number in the vertex form of a quadratic function affects its graph. End of Unit Problem 6 Note: May condense Lessons 15-16 for ACA
Checkpoint D	*Responding to Student Thinking* **Problem 1:** Press Pause: If students struggle to describe how graphs of quadratic functions differ based on the functions, make time to explore the idea further. For example, revisit the digital form of the activity referred to here to explore how changing the parameters influence the graph. **Problem 2:** Press Pause: If students struggle to identify the maximum or minimum of a quadratic function in vertex form, plan to make time to revisit the idea. For example, use the optional lesson referred to here to practice connecting the equation of the quadratic function with the graph that represents it.	
END OF UNIT ASSESSMENT		

Unit Title

Unit 3 (iM A1 U8): Quadratic Equations

Relevant Standards

Lesson	Standards	Lesson	Standards	Lesson	Standards
Lesson 1	HSA-CED.A.1	Lesson 9	HSA-REI.B.4.b HSA-SSE.B.3.a HSF-IF.C.8.a	Lesson 17	HSA-CED.A.1 HSA-REI.A HSA-REI.B.4.b HSF-IF.B.5
Lesson 2	HSA-REI.B.4 HSA-REI.B.4.b	Lesson 10	HSA-REI.B.4.b HSA-REI.D HSA-SSE.A HSA-SSE.A.2 HSF-IF.B.4 HSF-IF.C.8.a	Lesson 18	HSA-CED.A.1 HSA-REI.B.4.b HSF-IF.A.2
Lesson 3	HSA-REI.A.1 HSA-REI.B.4.b	Lesson 11	HSA-REI.B.4.b HSA-SSE.A.2	Lesson 19	HSA-REI.B.4.a HSA-SSE.A.2
Lesson 4	HSA-REI.B.4	Lesson 12	HSA-REI.B.4.a HSA-REI.B.4.b HSA-SSE.A HSA-SSE.A.2	Lesson 20	HSA-REI.B.4.b HSF-IF.C.7.a HSN-RN.B.3
Lesson 5	HSA-REI.A.1 HSA-REI.B.4 HSA-REI.B.4.b HSA-REI.D HSA-REI.D.10	Lesson 13	HSA-REI.A HSA-REI.B.4.b	Lesson 21	HSA-REI.B.4.b HSN-RN.B HSN-RN.B.3
Lesson 6	HSA-SSE.A.2	Lesson 14	HSA-REI.B.4.a HSA-REI.B.4.b HSA-SSE.A.2	Lesson 22	HSA-SSE.A.2 HSA-SSE.B.3 HSA-SSE.B.3.b HSF-IF.C
Lesson 7	HSA-SSE.A.2	Lesson 15	HSA-REI.B.4.a HSA-REI.B.4.b HSA-REI.D	Lesson 23	HSA-SSE.B.3.b HSF-IF.C HSF-IF.C.9
Lesson 8	HSA-SSE.A.2	Lesson 16	HSA-REI.B.4.b HSA-SSE.A	Lesson 24	HSA-REI.B.4.b HSA-REI.C.7 HSF-IF.C.8.a

Essential Question(s):

- Why do we need different methods to solve quadratic equations?
- How can the same quadratic equation have 0, 1, or 2 solutions, and what does this tell us about real-world situations?

  What mathematical structures and patterns help us transform quadratic expressions to reveal important information?
- How do we determine when solutions to equations are rational versus irrational, and why does this matter?
- When is it worth completing the square instead of using other solving methods?
- How does the quadratic formula connect to and generalize other solving methods we've learned?
- What makes a quadratic model appropriate for a real-world situation, and how do we interpret solutions in context?
- How do different representations of the same quadratic relationship highlight different aspects of

Enduring Understanding(s):

- Multiple solution methods exist for quadratic equations, and strategic method selection depends on the form of the equation and the desired information.
- The structure of quadratic expressions determines their solvability and the nature of their solutions.
- Completing the square is both a solving technique and a method for transforming quadratic expressions to reveal maximum/minimum values.
- The quadratic formula represents a generalized method derived from the completing the square process and works for any quadratic equation.
- Quadratic equations can have 0, 1, or 2 real solutions, and this number connects to graphical features and real-world constraints.
- Different forms of quadratic expressions reveal different information: standard form shows y-intercept, factored form shows zeros, and vertex form shows maximum/minimum.
- Solutions to quadratic equations can be rational or irrational, and understanding the nature of these

<table>
<tr><td>a problem?</td><td>solutions helps in determining appropriate representations and approximations.
• Quadratic models effectively represent many real-world phenomena involving optimization, projectile motion, and area problems, but solutions must be interpreted within contextual constraints.</td></tr>
<tr><td>Demonstration of Learning:</td><td>Pacing for Unit</td></tr>
<tr><td>Section A Checkpoint
Section B Checkpoint
Mid-Unit Assessment
Section C Checkpoint
Section D Checkpoint
Section E Checkpoint
End-of-Unit Assessment</td><td>30 days (15 class periods)
Lesson Modifications: (from adaptation pack)
• Combine Lessons 1 and 2 (particularly if Unit 7 Lesson 8 was skipped). Focus on Lesson 2 Activity 1.
• Remove Lesson 10. This focuses on factoring when the coefficient of the squared term is greater than 1.
• Remove or condense Lessons 11–15 on completing the square, depending on the emphasis in your standards on completing the square.
• Remove Lessons 19–21.</td></tr>
<tr><td>Family Overview (link below)</td><td>Integration of Technology:</td></tr>
<tr><td>https://accessim.org/9-12-aga/algebra-1/unit-8?a=family</td><td>• Desmos Online Graphing Calculator</td></tr>
<tr><td>Unit-specific Vocabulary:</td><td>Aligned Unit Materials, Resources, and Technology (beyond core resources):</td></tr>
<tr><td>Factored form (of a quadratic expression), quadratic equation, standard form (of a quadratic expression), zero (of a function), zero product property, linear term, perfect square, completing the square, quadratic formula</td><td>• Desmos Online Graphing Calculator
• Pear Assessment (Edulastic)
• IL Classroom Adapted Resources</td></tr>
<tr><td>Opportunities for Interdisciplinary Connections:</td><td>Anticipated misconceptions</td></tr>
<tr><td>Science & Engineering
• Projectile Motion: The flying potato activity (Lesson 1) models object trajectories using $h(t) = -16t^2 + 80t + 64$
• Maximum Height Problems: Using vertex form to find optimal launch conditions in physics
• Arch Design: Parabolic shapes in bridge and building construction
• Population Growth: Quadratic models for limited growth scenarios in biology
• Antenna Design: Parabolic reflectors for communication systems
Business & Economics
• Ticket Sales Revenue: Lesson 2's concert revenue model R=(8−2x)(x+5)
• Maximum Profit: Using vertex form to determine optimal production levels
• Break-even Analysis: Finding when profit equals zero using quadratic equations
• Price-Demand Relationships: Quadratic revenue functions showing optimal pricing
• Manufacturing Costs: Quadratic cost functions with economies of scale
Arts & Recreation
• Frame Design: The framing problem (Lesson 1) connects to mat cutting and display optimization
• Basketball Trajectories: Parabolic paths of shot attempts in sports analysis
• Diving Trajectories: Height vs. time relationships (Lesson 24's diving context)
• Garden Planning: Maximum area problems for</td><td>• Missing both solutions when solving quadratic equations
• Errors in quadratic formula application (sign errors, arithmetic mistakes)
• Confusion about number of solutions (thinking all quadratics have 2 solutions)
• Factoring errors when rewriting expressions
• Completing the square mistakes
• Misunderstanding rational vs. irrational solutions
See teacher's guide for specific misconceptions aligned to each lesson</td></tr>
</table>

landscape design • Parabolic Art: Mathematical curves in sculpture and graphic design	
Connections to Prior Units	**Connections to Future Units**
Relevant Unit(s) to review: • Grade 7, Unit 6: Expressions, Equations, and Inequalities Essential prior concepts to engage with this unit: • Write equivalent expressions by combining like terms and applying the distributive property.	Unit 4 (iM A2 U2): Polynomial Functions • Introduces complex solutions (a ± bi) that become central in this unit and and factoring techniques extend to polynomial functions Mathematical Modeling Foundations • Emphasizes setting up equations from real contexts and interpreting solutions within constraints, preparing students for the increased applications focus throughout Algebra 2 Unit 5 (iM A2 U3): Rational Functions and Equations • Students develop flexibility in choosing solving methods (factoring vs. quadratic formula vs. completing the square) that transfers to solving rational equations

Differentiation through *Universal Design for Learning*

Engagement
- Connect quadratic patterns to engaging real-world contexts like projectile motion, ball tosses, and revenue optimization
- Compare quadratic growth to familiar linear and exponential patterns to build understanding and maintain interest
- Scaffold learning progression from concrete visual patterns to abstract function forms and equations
- Build on prior knowledge of linear and exponential functions while highlighting unique characteristics of quadratic growth
- Make learning goals transparent about understanding different forms of quadratic expressions and their graphical meaning
- Encourage mathematical discourse about how coefficients affect parabola shape and position

Representation
- Present quadratic functions through multiple sensory modalities using tables, graphs, equations, and physical motion contexts
- Ensure graphing technology accessibility and provide clear visual design for parabola key features
- Support vocabulary development for quadratic terminology (vertex, axis of symmetry, standard/factored/vertex form)
- Connect symbolic expressions to verbal descriptions and graphical representations of parabola transformations
- Activate prior knowledge by explicitly connecting quadratics to familiar function types and growth patterns
- Guide pattern recognition in quadratic sequences and highlight critical features that distinguish parabola characteristics

Action & Expression
- Provide multiple ways for students to demonstrate understanding through graphing technology, algebraic manipulation, or contextual interpretation
- Support planning strategies for choosing appropriate quadratic forms to reveal specific function properties
- Offer various tools including graphing calculators, interactive applets, and function transformation templates
- Scaffold executive functions by providing organizational frameworks for moving between standard, factored, and vertex forms
- Create opportunities for students to express mathematical thinking about parabola transformations and real-world modeling
- Guide students in monitoring progress as they interpret coefficients' effects on graph shape and position

Supporting Multilingual/English Learners

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

MLR2: Collect and Display
Used in Lesson 12 (Building Perfect Squares) to capture and organize student language about completing the square
MLR5: Co-Craft Questions
Used in Lesson 2 (Revenue from Ticket Sales) where students generate mathematical questions about quadratic situations
MLR6: Three Reads

Used in Lesson 23 (All the World's a Stage) to help students analyze complex mathematical text about quadratic applications
MLR7: Compare and Connect
Used in Lesson 1 (Representing the Framing Problem) where students compare different approaches to writing quadratic equations and connect mathematical representations
MLR8: Discussion Supports
Used extensively throughout multiple lessons (Lessons 2, 4, 6, 9, 12, 16, 23) to support rich mathematical discussions about quadratic equations, factoring, completing the square, and the quadratic formula

Sentence Frames and Stems
Section A
- The equation _____ represents this situation because ...
- The solution(s) to the equation is/are ____. The solution(s) represent(s) ...

Section B
- To factor the expression _____ given in standard form, first I _____, then I ...
- I know the equation _____ has _____ solution(s) because ...
- To find the solution(s) to the equation _____, first I _____, then I ...
- The solution(s) to the equation _____ is/are _____.

Section C
- The equation _____ can be more easily solved by completing the square because ...
- The solution(s) to the equation _____ is/are _____.
- I know _____ is a perfect square because ...
- To solve the equation _____ by completing the square, first I _____, then I ...

Section D
- The product/sum of the values _____ is rational/irrational because ...
- The equation _____ can be more easily solved by using the quadratic formula because ...
- The solution(s) to the equation _____ is/are _____.
- The solution(s) represent(s) _____ in this situation.

Section E
- To convert the equation _____ to vertex form, first I _____, then I ...
- Using the equation _____ in vertex form, I recognize the _____ (feature of the graph) is _____ which represents _____.
- The solution(s) to the equation _____ is/are _____.

Section F
- I chose to write the quadratic equation in _____ form because ...
- Using the graph, the _____ (feature of the graph) is _____ and represents _____.
- Without graphing, I can tell _____ about a situation when the equation is in _____ form because ...

Related CELP standards and Learning Targets:

	Emerging	Expanding	Bridging
LT 1	Identify the key quantities and match a given quadratic equation to its described real-world situation. Use a word bank to label the meaning of the solutions.	Describe in short, connected sentences how the solutions (roots) of a given equation relate to the starting, ending, and maximum/minimum points of a simple context (e.g., a ball's flight).	Write a quadratic equation to model a described complex situation and justify the meaning of the solutions (real or non-real) in the context of the problem, using academic vocabulary.
LT 2	Use provided graphic organizers or manipulatives (e.g., algebra tiles) to factor the standard form of a quadratic expression, identifying the correct factors.	Explain the steps used to factor a quadratic expression and correctly create the equivalent expression in factored form with minor support.	Analyze a given quadratic expression and systematically generate the equivalent factored form, justifying the choice of factoring method used.
LT 3	Select the correct solutions (roots) from a multiple-choice list when given a quadratic equation already in factored form. Complete a sentence frame to state the zero-product property.	Explain in simple, ordered steps how to use the zero-product property to find the solutions to a factored quadratic equation.	Apply the zero-product property to solve a multi-step quadratic equation, explaining the logical connection between the equation's factors and the resulting solutions.
LT 4	Match the +/- symbol to the concept of "two solutions" and the square root symbol to the concept of "only one positive solution."	Compare and contrast the meaning and necessary use of the +/-symbol versus the square root symbol in mathematical contexts, using short comparative sentences.	Justify the mathematical reasoning for why the +/- symbol is required when solving by taking the square root, but not when interpreting the radical symbol, using precise mathematical language.

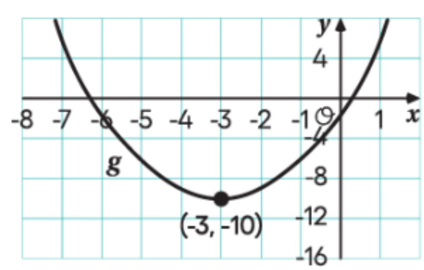
LT5	Follow a provided step-by-step example, labeling each step and identifying the missing number to create a perfect square trinomial (when b is even).	Describe and execute the steps for solving a quadratic equation by completing the square, using mathematical action verbs.	Evaluate and solve any quadratic equation by completing the square, explaining the transformation of the equation and the rationale for rearranging terms.
LT6	Classify given numbers as rational or irrational using a checklist. Use a provided rule (e.g., Rational + Irrational = Irrational) to state the result of a single operation.	Describe, using simple language and one numerical example for each, the general rules for the sum and product of a rational number and an irrational number.	Justify with clear mathematical evidence whether a given statement about the closure properties of rational and irrational numbers under addition and multiplication is true or false.
LT7	Identify and label the values of a, b, and c in a quadratic equation. Match the labeled values to the correct placement within the structure of the quadratic formula template.	Describe the purpose of the quadratic formula and demonstrate its use to solve a given quadratic equation, clearly showing the substitution and calculation steps.	Analyze the structure of the quadratic formula and use it to solve an equation, interpreting the discriminant to predict the number and type of solutions before fully solving.
LT8	Identify the vertex coordinates (h, k) from an equation written in vertex form $y=a(x-h)^2+k$. Select whether the vertex is a maximum or minimum from two choices.	Describe the process for finding the vertex of a quadratic function (using $-b/2a$ or completing the square) and explain how the vertex relates to the function's maximum or minimum value.	Rewrite a quadratic expression from standard form to vertex form (by completing the square) and use the vertex form to mathematically justify the maximum or minimum value in a practical context.

Unit Description

In this unit, students interpret, write, and solve equations algebraically.

Previously, students represented quadratic functions using expressions, tables, and descriptions. They connected important features of graphs to standard, factored, and vertex forms and expanded expressions into standard form.

Students begin with solving quadratic equations through reasoning without much algebraic manipulation. Then, they examine solving using the zero product property for quadratic equations that can be written in factored form. They notice patterns that help them rewrite quadratic expressions in factored form and recognize that not all of them are easily factorable.



This motivates finding another process of solving quadratic equations. Students recognize the benefits of equations of the form and develop the method of completing the square. Then, they generalize this process to the quadratic formula.

Throughout the unit, students analyze quadratic equations to recognize that there can be 0, 1, or 2 solutions. Solutions can be rational, irrational, or combinations of these. Students interpret the solutions that arise in different contexts.

The unit concludes with rewriting quadratic expressions from standard form into vertex form to find maximum and minimum values, then apply all of their understanding to applied problems.

Lesson Sequence	Learning Target	Success Criteria/ Assessment
Section A Finding Unknown Inputs Lessons 1-2	**Learning Target #1** Write quadratic equations, and reason about their solutions in terms of a situation.	**Lesson 1 Finding Unknown Inputs** I can explain the meaning of a solution to an equation in terms of a situation. • I can write a quadratic equation that represents a situation. Checkpoint A Problem 1 **Lesson 2 When and Why Do We Write Quadratic Equations?** • I can recognize the factored form of a quadratic expression and know when it can be useful for solving problems.

		● I can use a graph to find the solutions to a quadratic equation but also know its limitations.
Checkpoint A	*Responding to Student Thinking* **Problem 1:** Points to Emphasize - If students struggle to determine reasonable solutions from a situation, revisit the idea throughout the unit. For example, in the activity referred to here, when students find a solution, ask them about its meaning in the situation and whether it is reasonable.	
Section B Solving Quadratic Equations Lessons 3-10	**Learning Target #2** Given a quadratic expression of the form $ax^2 + bx + c$, create an equivalent expression in factored form. **Learning Target #3** Use factored form and the zero-product property to solve quadratic equations.	**Lesson 3 Solving Quadratic Equations by Reasoning** ● I can find solutions to quadratic equations by reasoning about the values that make the equation true. ● I know that quadratic equations may have two solutions. Mid-Unit Assessment Problem 1 **Lesson 4 Solving Quadratics Equations with the Zero-Product Property** ● I can explain the meaning of the "zero product property." ● I can find solutions to quadratic equations when one side is a product of factors and the other side is zero. Checkpoint B Problem 1 **Lesson 5 How Many Solutions?** ● I can explain why dividing by a variable to solve a quadratic equation is not a good strategy. ● I know that quadratic equations can have no solutions and can explain why when there are none. Mid-Unit Assessment Problem 2- 7c **Lesson 6 Rewriting Quadratic Expressions in Factored Form (Part 1)** ● I can explain how the numbers in a quadratic expression in factored form relate to the numbers in an equivalent expression in standard form. ● When given quadratic expressions in factored form, I can rewrite them in standard form. ● When given quadratic expressions in the form of ● $x^2 + bx + c$, I can rewrite them in factored form. Checkpoint B Problem 2 **Lesson 7 Rewriting Quadratic Expressions in Factored Form (Part 2)** ● I can explain how the numbers and signs in a quadratic expression in factored form relate to the numbers and signs in an equivalent expression in standard form. ● When given a quadratic expression given in standard form with a negative constant term, I can write an equivalent expression in factored form. Mid-Unit Assessment Problems 3-4 **Lesson 8 Rewriting Quadratic Expressions in Factored Form (Part 3)** ● I can explain why multiplying a sum and a difference, $(x + m)(x - m)$, results in a quadratic expression with no linear term. ● When given quadratic expressions in the form of $x^2 + bx + c$, I can rewrite them in factored form.

		Lesson 9 Solving Quadratic Equations by Using Factored Form • I can rearrange a quadratic equation to be written as an expression in factored form equal to zero and find the solutions. • I can recognize the number of solutions for a quadratic equation from the factored form. Mid-Unit Assessment Problem 6-7a,b ACC **Lesson 10 Rewriting Quadratic Expressions in Factored Form (Part 4)** • I can use the factored form of a quadratic expression or a graph of a quadratic function to answer questions about a situation. • When given quadratic expressions of the form • $ax^2 + bx + c$, where a is not 1, I can write equivalent expressions in factored form. Mid-Unit Assessment Problem 5
Checkpoint B	*Responding to Student Thinking* **Problem 1:** More Chances - Students will have more opportunities to develop this understanding in later sections. There is no need to slow down or add additional work to review this concept at this time. **Problem 2:** Points to Emphasize - If students struggle to factor quadratic expressions in standard form, provide additional practice throughout the rest of the unit. For example, the practice problems referred to here can help draw the connection between factored forms and standard forms. Algebra 1, Unit 8, Lesson 13, Practice Problem 5 Algebra 1, Unit 8, Lesson 15, Practice Problem 5	
Mid-Unit Assessment		
Section C Completing the Square Lessons 11-15	**Learning Target #4** Comprehend that the plus-minus symbol is used to represent both square roots of a number and that the square root notation expresses only the positive square root. **Learning Target #5** Solve quadratic equations of the form $x^2 + bx + c = 0$ by rearranging terms and completing the square.	**Lesson 11 What are Perfect Squares?** • I can recognize perfect-square expressions written in different forms. • I can recognize quadratic equations that have a perfect-square expression and solve the equations. End-of-Unit Assessment Problem 1 **Lesson 12 Completing the Square (Part 1)** • I can explain what it means to "complete the square" and describe how to do it. • I can solve quadratic equations by completing the square and finding square roots. Checkpoint C Problem 1 **Lesson 13 Completing the Square (Part 2)** • When given a quadratic equation in which the coefficient of the squared term is 1, I can solve it by completing the square. End-of-Unit Assessment Problem 4a **Lesson 14 Completing the Square (Part 3)** • I can complete the square for quadratic expressions of the form $ax^2 + bx + c$, where a is not 1, and explain the process. • I can solve quadratic equations in which the squared term coefficient is not 1 by completing the square. End of Unit Problem 4b **Lesson 15 Quadratic Equations with Irrational Solutions** • I can use the radical and "plus-minus" symbols to represent solutions to quadratic equations. • I know why the plus-minus symbol is used when

		solving quadratic equations by finding square roots. Checkpoint C Problem 2
Checkpoint C	*Responding to Student Thinking* **Problem 1:** Points to Emphasize - If students struggle to complete the square to solve quadratic equations, emphasize the idea throughout the next section. For example, return to the idea of completing the square whenever the quadratic formula is used to remind students of the connection. The lesson referred to here shows a connection. Algebra 1, Unit 8, Lesson 19 Deriving the Quadratic Formula **Problem 2:** Points to Emphasize: If students struggle to understand ± notation, emphasize its meaning throughout the next section. For example, in the lesson referred to here, write the quadratic formula three times: once using the plus-minus sign, then again, splitting it into the two forms, one with only a plus sign and one with only a minus sign. Algebra 1, Unit 8, Lesson 16 The Quadratic Formula	
Section D The Quadratic Formula Lessons 16-21	**Learning Target #6** Explain whether the products or sums of combinations of rational and irrational numbers are rational or irrational. **Learning Target #7** Use the quadratic formula to solve quadratic equations of the form $ax^2 + bx + c = 0$.	**Lesson 16 The Quadratic Formula** • I can use the quadratic formula to solve quadratic equations. • I know some methods for solving quadratic equations can be more convenient than others. Checkpoint D Problem 1 **Lesson 17 Applying the Quadratic Formula (Part 1)** • I can use the quadratic formula to solve an equation and interpret the solutions in terms of a situation. **Lesson 18 Applying the Quadratic Formula (Part 2)** • I can identify common errors when using the quadratic formula. • I know some ways to tell if a number is a solution to a quadratic equation. **ACC Lesson 19 Deriving the Quadratic Formula** • I can explain the steps and complete some missing steps for deriving the quadratic formula. • I know how the quadratic formula is related to the process of completing the square for a quadratic equation $ax^2 + bx + c = 0$. End-of-Unit Assessment Problem 5 **ACC Lesson 20 Rational and Irrational Solutions** • I can explain why adding a rational number and an irrational number produces an irrational number. • I can explain why multiplying a rational number (except 0) and an irrational number produces an irrational number. • I can explain why the sum or product of two rational numbers is rational. Checkpoint D Problem 2 **ACC Lesson 21 Sums and Products of Rational and Irrational Numbers** • I can explain why adding a rational number and an irrational number produces an irrational number. • I can explain why multiplying a rational number (except 0) and an irrational number produces an irrational number. • I can explain why the sum or product of two rational numbers is rational. End-of-Unit Assessment Problem 6
Checkpoint D	*Responding to Student Thinking*	

<table>
<tr><td></td><td colspan="2">Problem 1: Points to Emphasize - If students struggle to use the quadratic formula to solve equations, make time to provide additional practice throughout the rest of the unit. For example, revisit the practice problems referred to here to provide additional chances to use the formula.
Algebra 1, Unit 8, Lesson 19, Practice Problems 1 and 5
Problem 2: Points to Emphasize - If students struggle to classify values as rational or irrational, make time to revisit the idea. For example, after the Warm-up referred to here, revisit this problem, and use the Lesson Summary from the lesson also referred to here to match the values given in this checkpoint question to the bulleted statements about sums and products.
Algebra 1, Unit 8, Lesson 22, Activity 1 Three Expressions, One Function
Algebra 1, Unit 8, Lesson 21 Sums and Products of Rational and Irrational Numbers</td></tr>
<tr><td>Section E
Vertex Form Revisited

Lessons 22-23</td><td>Learning Target #8
Rewrite a quadratic expression in vertex form to identify the maximum or minimum value of the function that the expression defines.</td><td>Lesson 22 Rewriting Quadratic Expressions in Vertex Form
● I can identify the vertex of the graph of a quadratic function when the expression that defines it is written in vertex form.
● I know the meaning of the term “vertex form” and can recognize examples of quadratic expressions written in this form.
● When given a quadratic expression in standard form, I can rewrite it in vertex form.
Checkpoint E Problem 1
Lesson 23 Using Quadratic Expressions in Vertex Form to Solve Problems
● I can find the maximum or minimum of a function by writing, in vertex form, the quadratic expression that defines it.
● When given a quadratic function in vertex form, I can explain why the vertex is a maximum or minimum.
Checkpoint E Problem 1
End of Unit Problem 3</td></tr>
<tr><td>Checkpoint E</td><td colspan="2">Responding to Student Thinking
Problem 1: Points to Emphasize - If students struggle to find the vertex of the quadratic functions, plan to emphasize the idea during the lesson referred to here. For example, spend additional time discussing methods for finding the vertex, including completing the square to rewrite the equations in vertex form as well as using symmetry arguments.
Algebra 1, Unit 8, Lesson 24, Activity 2 The Dive</td></tr>
<tr><td>Section F
Let’s Put It To Work

Lesson 24</td><td>There are no new goals for this section.</td><td>ACC Lesson 24 Using Quadratic Equations to Model Situations and Solve Problems
● I can interpret information about a quadratic function given its equation or a graph.
● I can rewrite quadratic functions in different but equivalent forms of my choosing and use that form to solve problems.
● In situations modeled by quadratic functions, I can decide which form to use depending on the questions being asked.
End-of-Unit Assessment Problem 7</td></tr>
<tr><td colspan="3">End of Unit Assessment</td></tr>
</table>

Unit Title
Unit 4 (iM A2 U2): Polynomials
Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards	Lesson	Standards
Lesson 1	HSA-CED.A.2 HSA-SSE.A.1.a HSF-IF.A.2 HSF-IF.B.4 HSF-IF.B.5	Lesson 6	HSA-APR.A HSA-APR.A.1 HSA-APR.B HSA_SSE.B.3 HSF-IF.A.2	Lesson 11	HSA-REI.C.7 HSA-REI.D.11
Lesson 2	HSA-APR.A.1 HSA-CED.A.2 HSA-REI.D.11 HSA-SSE.A.1 HSF-IF.A.2	Lesson 7	HSA-APR.B HSA-SSE.A.1.a	Lesson 12	HSA-APR.A HSA-APR.B.3 HSF-IF.C.7.c
Lesson 3	HSA-SSE.A HSF-IF.C.7 HSF-IF.C.9	Lesson 8	HSA-SSE.A	Lesson 13	HSA-APR.A
Lesson 4	HSA-APR.A.1	Lesson 9	HSA-SSE.A HSF-IF.C	Lesson 14	HSA-APR.A HSA-APR.B HSA-APR.B.3 HSF-IF.C.7.c
Lesson 5	HSA-APR.B HSA-APR.B.3 HSF-IF.C.9	Lesson 10	HA-APR.B.3 HSF-IF.C.7.c	Lesson 15	HSA-APR.B.2

Essential Question(s)	Enduring Understanding(s)
• Why is it useful to write polynomials in factored form versus standard form? • What can we learn from a polynomial's factors that we can't see from its expanded form? • When is each form most advantageous for solving problems? • How are polynomial operations similar to operations with whole numbers? • Why do the same rules for multiplication and division apply to both polynomials and integers? • How can we predict the x-intercepts of a polynomial from its factored form? • What does it mean when a zero has "multiplicity," and how does this affect the graph? • Why does (x−a) being a factor guarantee that a is a zero? • What determines the end behavior of a polynomial function? • How can we sketch a polynomial's graph without plotting many points? • What role do degree and leading coefficient play in a polynomial's behavior? • What types of situations can be modeled by polynomials of different degrees? • How do we determine a reasonable domain for polynomial models? • What are the limitations of polynomial models? • What does the Remainder Theorem tell us about factors and zeros?	Polynomial functions can be represented in multiple equivalent forms (standard form, factored form), and each form reveals different structural information that helps us understand the function's behavior, including its zeros, intercepts, end behavior, and overall shape.

<table>
<tr><td>

- How is polynomial division similar to and different from numerical division?
- When is it useful to divide polynomials in problem-solving contexts?

</td><td></td></tr>
<tr><th>Demonstration of Learning</th><th>Pacing for Unit</th></tr>
<tr><td>
CFA 1 Checkpoint A (Lessons 1-4)
CFA 2 Checkpoint B (Lessons 5-7)
CFA 3 Checkpoint C (Lessons 8-11)
NOTES

- Question 1 replace written end behavior with notation instead of words “as x->infinity…
- Remove question 3

CFA 4 Checkpoint D (Lessons 12-15)
End of Unit Assessment
NOTES

- Problem 1: change to drop down menus for different parts (degree - odd/even)(constant - positive/negative)(end behavior - written using notation instead of words “as x->infinity, etc…)
- Problem 2: remove answer choice E - change to SAT style question
- Problem 3: remove this question (not a key concept for this unit) - Change to “rewrite the following polynomial in standard form: y = (x + 2)(x - 9)(x + 6)”
- Problem 5: Change the equation to say “ + d” instead of “ - d” Also change the equation to have d come out to a whole number
- Problem 6: For academic only, change the given factor of (x-3) to (2x+1)
- Problem 7: Change equation so that the (3x+4) changes to (2x +5), then reorder the question: start with part b (change to x- and y- intercepts), then part c (give drop down box for end behavior), then part a (graph)

</td><td>
20 days (10 class periods)
Lesson Modifications: (from adaptation pack)

- No lessons to remove or merge

</td></tr>
<tr><th>Family Overview (link below)</th><th>Integration of Technology:</th></tr>
<tr><td>https://accessim.org/9-12-aga/algebra-2/unit-2?a=family</td><td>

- Desmos Online Graphing Calculator

</td></tr>
<tr><th>Unit-specific Vocabulary</th><th>Aligned Unit Materials, Resources, and Technology (beyond core resources)</th></tr>
<tr><td>polynomial, degree, relative maximum, relative minimum, end behavior, multiplicity</td><td>

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- IL Classroom Adapted Resources
- Blank paper
- Chart paper
- Desmos graphing calculator
- Math Community Chart
- Rulers
- Scientific calculators
- Scissors
- Slips of paper
- Tape

</td></tr>
<tr><th>Opportunities for Interdisciplinary Connections:</th><th>Anticipated misconceptions</th></tr>
<tr><td>
CTE

- Investment growth modeling
- Profit maximization
- Market analysis
- Break-even analysis
- Package design optimization

Science (Physics, Environmental)

- Projectile motion

</td><td>

- Degree confusion: Saying $5x^2+3x^4-2$ has degree 2 instead of degree 4
- Zeros from factors: Not connecting (x-3) as a factor to x = 3 as a zero
- Sign errors: Thinking (x + 4) gives a zero at x = 4 instead of x = -4
- Multiplicity confusion: Not recognizing $(x-2)^3$ creates

</td></tr>
</table>

- Wave behavior - Optics - Population dynamics	different graph behavior than (x- 2) - End behavior patterns: Thinking all polynomial graphs "go up" on both ends. - Not connecting odd degree → opposite ends, even degree → same ends • Leading coefficient impact: Ignoring how negative leading coefficients affect end behavior • Intercept vs. zeros: Confusing x-intercepts (points) with zeros (values) See teacher's guide for specific misconceptions aligned to each lesson
Connections to Prior Units	**Connections to Future Units**
Relevant Unit(s) to review: - Algebra 1, Unit 2 and Unit 6 - Algebra 1, Unit 7, Lesson 8 Essential prior concepts to engage with this unit - Multiplying binomials. - Solving systems of linear equations by substitution.	Unit 5 (iM A2 U3): Rational Functions and Equations - End behavior analysis from polynomial functions directly applies to understanding asymptotic behavior in rational functions - Factoring polynomial expressions is essential for simplifying rational expressions and finding vertical asymptotes - Polynomial division (Remainder Theorem) connects to analyzing rational function behavior and finding holes vs. asymptotes Unit 6 (iM A2 U4): Complex Numbers and Rational Exponents - Polynomial zeros and factors extend to finding complex zeros using the Fundamental Theorem of Algebra - Factoring techniques learned with real polynomials apply to factoring over the complex numbers - Polynomial degree and multiplicity concepts transfer to understanding complex solutions

Differentiation through *Universal Design for Learning*

Engagement
- Connect polynomial patterns to familiar contexts and extend understanding from linear and quadratic functions
- Compare polynomial growth rates to previously learned function types to maintain interest and build understanding
- Scaffold learning progression from concrete graphical features to abstract algebraic operations and polynomial division
- Build on prior knowledge of factoring and multiplication while highlighting connections to zeros and factors
- Make learning goals transparent about understanding polynomial forms and their relationship to graphical features
- Encourage mathematical discourse about end behavior, multiplicity effects, and the Remainder Theorem

Representation
- Present polynomials through multiple sensory modalities using graphs, factored forms, standard forms, and division algorithms
- Ensure graphing technology accessibility and provide clear visual design for polynomial key features like zeros and end behavior
- Support vocabulary development for polynomial terminology (degree, leading coefficient, multiplicity, end behavior, Remainder Theorem)
- Connect symbolic polynomial expressions to verbal descriptions and graphical representations of function behavior
- Activate prior knowledge by explicitly connecting polynomial operations to familiar number operations and distributive property
- Guide pattern recognition in polynomial graphs and highlight critical features that distinguish degree and multiplicity effects

Action & Expression
- Provide multiple ways for students to demonstrate understanding through factoring, graphing, division algorithms, or contextual interpretation
- Support planning strategies for choosing appropriate polynomial forms to reveal specific function properties like zeros or end behavior
- Offer various tools including graphing technology, polynomial division templates, and factoring organizers
- Scaffold executive functions by providing organizational frameworks for moving between standard and factored forms
- Create opportunities for students to express mathematical thinking about factor-zero relationships and polynomial behavior
- Guide students in monitoring progress as they connect algebraic operations to graphical features and apply the Remainder Theorem

Supporting Multilingual/English Learners

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

MLR1: Stronger and Clearer Each Time

Used in Lesson 10 (Sketching Polynomials) where students refine their explanations of how to sketch polynomial graphs from factored form

MLR2: Collect and Display

Used extensively in multiple lessons

Sentence Frames and Stems

Section A

- The degree of the polynomial _____ is _____ because ...
- The constant term of the polynomial _____ is _____.
- The point _____ is the relative minimum/maximum on the graph because ...
- A reasonable domain for the polynomial function _____ is _____ because ...
- The function _____ represents this situation because ...
- When the polynomial _____ is combined with the polynomial _____ using _____ (operation), the result is/is not a polynomial because ...

Section B

- The expression _____ represents the polynomial _____ with zeros at ...
- The zeros of the polynomial _____ are _____ because ...
- The degree of the polynomial _____ is _____ because ...
- The constant term of the polynomial _____ is _____.
- The polynomial _____ in standard/factored form is _____ in factored/standard form.

Section C

- To sketch the graph of the polynomial _____, first I _____, then I ...
- To find the point(s) of intersection for the polynomials, first I _____, then I ...
- The system of equations has _____ solution(s). I know this because ...
- The solution to the system of equations is _____ because ...
- _____ is a possible equation for a polynomial with the following properties: ...
- As _____ gets _____ in the _____ direction, _____ gets _____ in the _____ direction.

Section D

- The polynomial _____ with a known factor of _____, can be written as a product of the linear factors _____ (a set of 2 or factors)because ...
- To divide the polynomial _____ by the factor _____, first I _____, then I ...
- I chose to use _____ (method) to write the polynomial _____ in factored form because ...

Supporting Multilingual/English Learners (*CELP standards*)

	Emerging	Expanding	Bridging
LT1	I can identify the dimensions (length, width, height) of a box and recognize that volume = length x width x height with visual support and sentence frames like "The volume is _____ times _____ times _____."	I can write volume expressions like V=x(x-2)(x+3) and explain what each factor represents using mathematical language with some support: "The factor (x-2) represents the width because..."	I can independently create volume expressions, interpret what restrictions on xxx make sense in context, and explain why certain values are reasonable: "Since x represents length, x>2 because width must be positive."
LT2	I can match polynomial expressions to real-world contexts (area, volume, profit) using visual aids and choose from given options like "This polynomial could represent: A) area B) number of people C) temperature"	I can explain connections between polynomial features and context with sentence frames: "The degree tells us..." "The coefficient means..." and identify which situations polynomials can and cannot model.	I can analyze polynomial models critically, compare different polynomial models for the same situation, and justify why polynomial models are appropriate or inappropriate for specific contexts.
LT3	I can locate and name basic features (x-intercepts, y-intercepts, maximum/minimum points) using vocabulary cards and sentence frames	I can describe relationships between algebraic and graphical features using mathematical language: "The factor (x-3) creates an x-intercept at..." "The	I can analyze and compare multiple representations fluently, explaining connections: "The multiplicity of the zero affects the graph's behavior

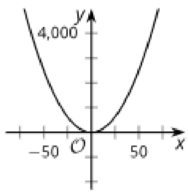
	like "The x-intercept is at x=" "The graph increases/decreases."	leading coefficient affects..."	because..." and make predictions about unseen features.
LT4	I can perform polynomial operations with step-by-step guidance and recognize the result is still a polynomial using sentence frames: "When I add these polynomials, I get..." "The result is still a polynomial because..."	I can explain why polynomial operations result in polynomials using mathematical language: "The degree of the sum is..." "Polynomial closure means..." and connect this to the definition of polynomials.	I can generalize the closure property, explain exceptions (like division), and apply this understanding to determine if expressions are polynomials: "This expression is/isn't a polynomial because of the closure property..."
LT5	I can identify zeros by setting each factor equal to zero using sentence frames: "If (x-3) is a factor, then x = ___" and visual supports showing the connection between factors and x-intercepts.	I can find zeros from factored form and explain the process using mathematical language: "The zeros are found by..." and connect zeros to x-intercepts on graphs with guided support.	I can find zeros from complex factored forms, explain the relationship between factors and zeros, and analyze how multiplicity affects the graph at each zero.
LT6	I can identify basic features (degree, leading coefficient, x-intercepts) from standard and factored forms using vocabulary cards and sentence frames: "The degree is ___" "The leading coefficient is ___"	I can compare how standard and factored forms reveal different information using mathematical language: "Standard form shows..." "Factored form reveals..." and explain advantages of each form.	I can analyze polynomial features across multiple representations, strategically choosing forms based on needed information, and explaining complex relationships between algebraic and graphical features.
LT7	I can write simple polynomial expressions with given x-intercepts using patterns: "If zeros are 2 and -1, then factors are (x-2) and (x+1)" with step-by-step guidance and visual supports.	I can construct polynomial functions with specified intercepts and explain my reasoning: "To have zeros at x = 3 and x = -2, I need factors..." using mathematical language with some support.	I can create polynomial functions with complex intercept conditions (including multiplicity), justify my methods, and explain how different constructions affect the graph's behavior.
LT8	I can identify the leading term and describe basic end behavior using sentence frames: "As x gets very large, y..." "As x gets very negative, y..." with visual graphs showing patterns.	I can explain why the leading term determines end behavior using mathematical language: "The leading term dominates because..." and predict end behavior from equations with guided support.	I can analyze end behavior in complex situations, explain the mathematical reasoning behind leading term dominance, and compare end behavior across different polynomial types.
LT9	I can plot zeros on a graph and identify whether the graph crosses or touches the x-axis using visual patterns and sentence frames: "At x = 2, the graph crosses/touches because the multiplicity is ___"	I can sketch polynomial graphs using zeros and multiplicities, explaining the relationship: "When multiplicity is even, the graph touches..." "When multiplicity is odd, the graph crosses..." with mathematical language.	I can create accurate polynomial graphs considering zeros, multiplicities, and end behavior, explaining complex behaviors and making connections between algebraic features and graphical characteristics.
LT10	I can find where two simple polynomial functions intersect by setting them equal and solving with step-by-step support, using sentence frames: "To find intersections, I set ___ equal to ___"	I can solve intersection problems for polynomial functions and explain my method: "The functions intersect when..." using mathematical language and connecting algebraic solutions to graphical intersections.	I can find intersections of complex polynomial functions, analyze the meaning of solutions in context, and explain why intersection points represent equation solutions.
LT11	I can perform basic polynomial division with step-by-step guides and identify quotient and remainder using vocabulary supports and sentence frames: "When I divide ___ by ___, I get quotient ___ and remainder ___"	I can divide polynomials using long division and explain the process using mathematical language: "First, I divide the leading terms..." connecting division to multiplication relationships.	I can perform complex polynomial division, apply the division algorithm strategically, and explain how division relates to factoring and the Remainder Theorem.
LT12	I can use a known factor to find other factors through division with guided support, using sentence frames: "If (x-2) is a factor, then dividing gives me..." and visual step-by-step processes.	I can apply polynomial division to completely factor polynomials starting from one known factor, explaining the process: "After dividing by the known factor, I can factor the quotient..."	I can strategically use division and known factors to completely analyze polynomial functions, sketch graphs from factored forms, and explain the complete factorization process.
LT13	I can apply the Remainder Theorem using substitution with step-by-step guidance: "To find the remainder when dividing by (x-a), I substitute x = a" using sentence frames and examples.	I can explain the Remainder Theorem and why it works using mathematical language: "The remainder equals f(a) because..." and apply it to solve problems with guided support.	I can prove why the Remainder Theorem is true, apply it in complex situations, and connect it to broader concepts of polynomial division and function evaluation.

Unit Description

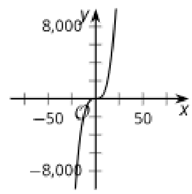
This unit extends students' previous work with linear and quadratic functions as they investigate polynomials of higher degree. Students rewrite polynomials in different forms, recognizing the benefits of the various forms for their ability to reveal the structure of key features of their graphs.

The unit begins with an introduction to two situations that can be modeled by a polynomial function. Students build their understanding of what polynomials are and what their graphs can look like. Certain aspects, such as end behavior, will be important in a later unit when students explore the end behavior of rational functions.

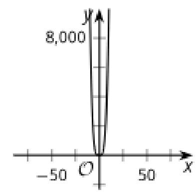
$y = x^2$



$y = x^3$



$y = x^4$



Focusing on functions expressed in factored form and their graphs, students connect that a factor of $(x - a)$ means a is a zero of the function and $(a, 0)$ is a horizontal intercept. The effect of the degree and leading coefficient on end behavior is established along with the effect of multiplicity on the shape of the graph near zeros of the function. Taking in all of these features, students learn to sketch polynomial functions expressed as a product of linear factors.

In a previous course, students used the distributive property to multiply factors, and also factored quadratics. Opportunities to review these skills and apply them to polynomials of higher degree are embedded throughout the unit and in practice problems. This prepares students for the final section where students divide a polynomial written in standard form by a suspected factor. From there, the connection between division and multiplication equations is used to establish the Remainder Theorem. This allows the conclusion that if a polynomial has a zero at $x = a$, then it must also have $(x - a)$ as a factor.

Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
Section A What Is a Polynomial? Lessons 1-4	**Learning Target 1** Generalize that when polynomials are combined by addition, subtraction, or multiplication, the result is a polynomial. **Learning Target 2** Identify and interpret relative minimums, relative maximums, the degree, and the constant term of polynomials and their graphs.	**Lesson 1 Let's Make a Box** • I can create and interpret an expression that models the volume of a box. **Lesson 2 Funding the Future** • I can use polynomials to understand different kinds of situations. **Lesson 3 Introducing Polynomials** • I can identify important characteristics of polynomial graphs and expressions. Checkpoint A Problem 1 End of Unit Problem 1 **Lesson 4 Combining Polynomials** • I understand that if you add, subtract, or multiply polynomials, you get another polynomial. Checkpoint A Problem 2 End of Unit Problem	
Checkpoint A	*Responding to Student Thinking:* **Problem 1**: More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time. **Problem 2:** Points to Emphasize: If most students struggle with the definition of a polynomial, focus on this as opportunities arise in the next several lessons. For example, in the Synthesis of the activity referred to here, ask students whether these are polynomial functions and to explain how they know.		
Section B Working with Polynomials	**Learning Target 3**	**Lesson 5 Connecting Factors and Zeros** • I can find the zeros of a function from its factored form.	

<table>
<tr><td>Lessons 5-7</td><td>Determine properties of polynomials by representing them in standard or factored form.

Learning Target 4
Generate a possible expression for a polynomial function given the horizontal intercepts of the function.

Learning Target 5
Identify zeros of polynomial functions written in factored form.</td><td>Checkpoint B Problem 1
Lesson 6 Different Forms
● I can identify features of polynomials and their graphs using their standard and factored forms.
End of Unit Problems 1 and 2
Lesson 7 Using Factors and Zeros
● I can write an expression for a function that has specific horizontal intercepts.
End of Unit Problems 2</td></tr>
<tr><td>Checkpoint B</td><td colspan="2">Responding to Student Thinking
Problem 1: Points to Emphasize: If most students struggle with finding the zeros of a polynomial from its factors, focus on this as opportunities arise over the next several lessons. For example, in the Activity Synthesis of this activity, spend time discussing how the factored form of the polynomials can be used to find where the value of y is 0.
Problem 2: Press Pause: By this point in the unit, there should be some student mastery of identifying the degree and constant term of a polynomial from its factored form. If most students struggle with these, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.</td></tr>
<tr><td>Section C
Graphs of Polynomials

Lessons 8-11</td><td>Learning Target 6
Calculate the solution to a system of polynomial equations.

Learning Target 7
Create equations for polynomial functions with specific end behavior.

Learning Target 8
Use zeros and multiplicities to sketch a graph of a polynomial function given in factored form.</td><td>Lesson 8 End Behavior (Part 1)
● I understand why a function’s end behavior is determined by its leading term.
End of Unit Problem 1 and 7
Lesson 9 End Behavior (Part 2)
● I can identify the end behavior of a polynomial function from its equation.
End of Unit Problem 1 and 7
Lesson 10 Multiplicity
● I can use zeros and multiplicities to sketch a graph of a polynomial.
End of Unit Problem 7
Lesson 11 Finding Intersections
● I can find where two polynomial functions intersect.
End of Unit Problem 4</td></tr>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.</td></tr>
<tr><td>Section D:
Polynomial Division

Lessons 12-15</td><td>Learning Target 9
Generalize that for a polynomial p(x) and a number a, the remainder on division by (x-a) is p(a).

Learning Target 10
Rewrite a polynomial as the product of linear factors.</td><td>Lesson 12 Polynomial Division (Part 1)
● I can divide one polynomial by another.
End of Unit Problem 5 and 6
Lesson 13 Polynomial Division (Part 2)
● I can use long division to divide polynomials.
End of Unit Problem 5 and 6
Lesson 14 What Do You Know about Polynomials?
● I can use division to rewrite a polynomial in factored form starting from a known factor and then sketch what it looks like.
End of Unit Problem 6
Lesson 15 The Remainder Theorem
● I understand the Remainder Theorem and why it's true.
End of Unit Problem 5</td></tr>
</table>

Checkpoint D	*Responding to Student Thinking* Press Pause: By this point in the unit, there should be some student mastery of using polynomial division. If most students struggle with this, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.
End of Unit Assessment	

Unit Title	
Unit 5 (iM A2 U3): Rational Functions and Equations	
Relevant Standards: Bold indicates priority	

Lesson	Standards	Lesson	Standards	Lesson	Standards
Lesson 1	HSA-CED.A HSA-CED.A.2 HSA-CED.A.4	Lesson 5	HSA-CED.A.1 HSA-REI.A HSA-CED.A.2 HSA-REI.A.1	Lesson 9	HSA-APR.D MP1
Lesson 2	HSA-CED.A.2 HSF-IF.A.2 HSF-IF.C HSF-IF.C.7 HSA-CED.A.4 HSF-BF.B.3 HSF-IF.B.4	Lesson 6	HSA-CED.A.1 HSA-REI.A.1 HSA-REI.D.11 MP1 MP2 MP5	Lesson 10	HSA-APR.C HSA-SSE.A.2 HSA-SSE.B.4 MP1 MP7
Lesson 3	HSA-APR.D.6 HSA-SSE.A.1 HSF-IF.C	Lesson 7	HSA-REI.A.2 MP6 MP7	Lesson 11	HSA-SSE.A.1 HSA-SSE.B.4 MP1 MP5
Lesson 4	HSF-IF.C.7 HSA-APR.D.6 HSA-SSE.A.1 HSA-SSE.A.1.c	Lesson 8	HSA-APR.C.4 HSA-SSE.A.2 HSA-SSE.B.4 MP7 MP8		

Essential Question(s):	Enduring Understanding(s):
• How do rational functions help us model and understand real-world optimization problems? • What stories do the asymptotes of rational functions tell about real-world situations? • When does an algebraic solution fail to solve the real-world problem it represents? • How can we distinguish between equations that are sometimes true and mathematical truths that are always true?	• The algebraic form of rational functions (degree relationships, factorization, restrictions) directly determines their graphical features like asymptotes and discontinuities • Rational functions naturally model inverse relationships and trade-offs in real-world optimization problems (cost per unit, surface area vs. volume) • Values that make rational expressions undefined represent physical impossibilities, economic constraints, or logical boundaries in context • Solving rational equations can produce extraneous solutions that satisfy manipulated equations but not the original problem • Algebraic, graphical, and contextual views each reveal different aspects requiring flexible movement between them

Demonstration of Learning:	Pacing for Unit
CFA 1: Checkpoint A (Lessons 1-4) CFA 2: Checkpoint B (Lessons 5-7) NOTES • For academic: Change equation to 1/(x-5)=x/(x-5) • Keep as is for accelerated CFA 3: (accelerated only) Checkpoint C (Lessons 8-11) Checkpoint C is for feedback only (for academic) End of Unit Assessment NOTES • Problem 1: Change the denominator to x + 3, then change the answers based on the new equation • Problem 2: remove for academic, keep for accelerated • Add additional solving rational equations to test for accelerated only (rational + rational = rational) or (find common factors then solve) - can be done on teacher	20 days (10 class periods) Lesson Modifications: (from adaptation pack) • ACA Can remove/skip Lessons 8-11 • Optional Lesson 2 may be safely skipped if the Check Your Readiness assessment indicates that students are familiar enough with the concepts to engage with the other lessons.

level • Problem 5: remove for academic, keep for accelerated: add part b “show that the expressions are equivalent” • Problem 6: split part b into parts b and c - part b “at what value of w does the graph have a vertical asymptote? Explain how you know.” part c “What does the vertical asymptote mean in the situation” • For academic: Add a question where students identify the equations of the vertical and horizontal asymptotes from a graph	
Family Overview (link below)	**Integration of Technology:**
https://accessim.org/9-12-aga/algebra-2/unit-3?a=family	• Desmos Online Graphing Calculator
Unit-specific Vocabulary:	**Aligned Unit Materials, Resources, and Technology (beyond core resources):**
Rational function, Rational expression, Rational equation, Vertical asymptote, Horizontal asymptote, End behavior, Equivalent forms, Extraneous solution, Domain restriction, Discontinuity, Hole (removable discontinuity)	• Desmos Online Graphing Calculator • Pear Assessment (Edulastic) • IL Classroom Adapted Resources
Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions**
Business & Economics • Average cost functions for manufacturing, break-even analysis, geometric series for loan payments, and supply/demand optimization Health & Medicine • Drug concentration and elimination rates, dosage optimization, epidemiological modeling, and medical imaging mathematics Environmental Science • Population dynamics and carrying capacity models, resource sustainability analysis, carbon footprint optimization, and climate modeling	• Domain and Asymptote Confusion • Invalid Algebraic Manipulation • Extraneous Solutions Ignored • Identity vs. Equation Confusion • Modeling Reality Disconnect - Students assume mathematical solutions always make contextual sense See teacher’s guide for specific misconceptions aligned to each lesson
Connections to Prior Units	**Connections to Future Units**
Relevant Unit(s) to review: • Algebra 1 Unit 6: Introduction to Exponential Functions Essential prior concepts to engage with this unit • Exponential functions • Rational exponents The optional lesson and previous units of this course should provide enough support for students to engage in the material for this unit. If the formative assessments provided in this unit indicate that additional resources are needed, consider using Algebra 1 Unit 6 for additional resources.	Unit 6 (iM A2 U4): Complex Numbers and Rational Exponents • Extraneous solutions analysis from rational equations extends to radical equations, where extraneous solutions are even more common • Equation-solving strategies developed in Unit 3 apply to equations with rational exponents • Domain restrictions understanding helps with radical function domains and complex number contexts
Differentiation through *Universal Design for Learning*	
Engagement • Connect rational functions to engaging real-world contexts like average costs, surface area minimization, and travel time calculations • Build on familiar polynomial functions by introducing rational functions as fractions with polynomials in numerator and denominator • Scaffold learning progression from concrete asymptotic behavior observations to abstract polynomial division techniques • Address misconceptions about extraneous solutions by connecting algebraic manipulation to graphical intersections • Make learning goals transparent about understanding asymptotes, end behavior, and the relationship between rational equations and their solutions • Encourage mathematical discourse about polynomial identities and their applications to geometric series formulas	

Representation

- Present rational functions through multiple sensory modalities using graphs showing asymptotes, tables, equations, and real-world contexts
- Ensure graphing technology accessibility and provide clear visual design for asymptotic behavior and discontinuities Support vocabulary development for rational function terminology (asymptote, discontinuity, end behavior, extraneous solution, polynomial identity)
- Connect symbolic rational expressions to verbal descriptions and graphical representations of function behavior and restrictions
- Activate prior knowledge by explicitly connecting polynomial division to familiar long division algorithms with numbers Guide pattern recognition in rational function graphs and highlight critical features like vertical and horizontal asymptotes

Action & Expression

- Provide multiple ways for students to demonstrate understanding through polynomial division, graphing, equation solving, or contextual modeling
- Support planning strategies for choosing appropriate methods to solve rational equations and identify potential extraneous solutions
- Offer various tools including graphing technology, polynomial division organizers, and step-by-step equation solving templates
- Scaffold executive functions by providing organizational frameworks for connecting algebraic manipulation to graphical verification
- Create opportunities for students to express mathematical thinking about asymptotic behavior and prove polynomial identities
- Guide students in monitoring progress as they apply geometric series formulas and verify solutions to rational equations

Supporting Multilingual/English Learners

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

MLR1: Stronger and Clearer Each Time

Used in Lesson 3 (Horizontal Asymptotes) where students refine their explanations of matching graphs to equations, Lesson 6 (Rational Resistance) where students improve descriptions of graphing strategies, and Lesson 7 (Rational Solving) where students refine explanations about extraneous solutions

MLR2: Collect and Display

Used in Lesson 2 (Biking 10 Miles Part 2) to capture and organize student language about rational functions and vertical asymptotes, and Lesson 3 (Publishing a Paperback) to collect language about horizontal asymptotes and end behavior

MLR5: Co-Craft Questions

Used in multiple lessons:

- Lesson 4 (Combined Fuel Economy) where students generate mathematical questions about rational functions in context
- Lesson 5 (Batting Averages) where students create questions about rational equations
- Lesson 6 (Two Directions) where students generate questions about boat travel contexts

MLR6: Three Reads

Used in Lesson 10 (Breakthrough Artist?) to help students analyze complex contextual problems involving geometric sequences and sums

MLR7: Compare and Connect

Used in Lesson 9 (Theorems and Identities) where students compare different strategies for proving polynomial identities, and Lesson 11 (Some Interesting Sums) where students compare different approaches to finding geometric sums

MLR8: Discussion Supports

Used extensively throughout multiple lessons (Lessons 1, 2, 4, 6, 7, 8, 9, 11) to support rich mathematical discussions about rational functions, asymptotes, rational equations, extraneous solutions, and polynomial identities

Sentence Frames and Stems

Section A

- The rational function _____ is helpful in this situation because ...
- The function _____ has/does not have a horizontal asymptote because ...
- The equation _____ represents the horizontal asymptote for the function _____.
- The asymptote _____ represents _____ in this situation because ...

- The end behavior of the function _____ can be written as _____.

Section B

- _____ is/are the solution(s) to the equation _____ because ...
- To solve the rational equation _____, first I _____, then I ...
- The expression _____ represents this situation because ...
- The equation _____ represents this situation because ...
- The equation _____ is equivalent to the equation _____ because ...

Section C

- The equation _____ is/is not an identity because ...
- The sum of the terms in the geometric sequence _____ (list terms) is _____.
- The situation can be modeled using a geometric sequence because ...
- I used the formula _____ to find that the total number of _____ after _____ was/is _____.

Related *CELP standards* and Learning Targets:

	Emerging	Expanding	Bridging
LT 1	I can identify the parts of rational functions that model cylinder properties using visual supports and sentence frames: "The numerator represents ___" "The denominator represents ___" with guided examples.	I can create rational functions to model cylinder situations and explain what each part means: "This function models surface area because..." using mathematical language with some support.	I can develop and analyze complex rational function models for cylinders, justify my modeling choices, and explain limitations and assumptions in real-world contexts.
LT 2	I can locate vertical asymptotes on graphs and find them in equations by setting the denominator equal to zero using sentence frames: "The vertical asymptote is at x = ___" "The denominator equals zero when ___"	I can identify vertical asymptotes from graphs and equations and explain why they occur: "Vertical asymptotes happen when the denominator is zero because..." using mathematical language.	I can analyze vertical asymptotes in complex rational functions, distinguish between asymptotes and holes, and explain the mathematical reasoning behind asymptotic behavior.
LT 3	I can find horizontal asymptotes by comparing degrees of numerator and denominator using visual patterns and sentence frames: "When the degrees are equal, the horizontal asymptote is y = ___"	I can determine horizontal asymptotes and explain the degree comparison rules: "The horizontal asymptote depends on the degrees because..." connecting to end behavior concepts.	I can analyze horizontal asymptotes in complex situations, explain why degree relationships determine asymptotes, and connect to limit behavior of rational functions.
LT 4	I can rewrite simple rational functions in the form f(x) = q(x) + r(x)/b(x) with step-by-step guidance and identify the end behavior using sentence frames: "As x gets large, f(x) approaches ___"	I can perform polynomial division to rewrite rational functions and explain how this reveals end behavior: "The quotient tells us..." "The remainder term approaches zero because..."	I can strategically rewrite complex rational functions to analyze end behavior, explain the mathematical reasoning behind the approach, and connect to asymptotic analysis.
LT5	I can write simple rational expressions for averages using templates and sentence frames: "The average is total divided by number, so the expression is ___" with visual supports showing the relationship.	I can create rational expressions representing averages in various contexts and explain my reasoning: "This expression represents the average because..." using mathematical language.	I can develop and analyze complex average rate models using rational expressions, interpret results in context, and explain how changing conditions affect the average.
LT6	I can solve simple rational equations by finding common denominators with step-by-step guidance using sentence frames: "First, I multiply both sides by ___" "Then I solve ___"	I can solve rational equations and explain my solution method: "I multiply by the LCD because..." "This step eliminates fractions because..." using mathematical language with support.	I can solve complex rational equations using strategic methods, explain why different approaches work, and connect solution processes to function properties.
LT7	I can check solutions by substituting back into the original equation and identify when answers don't work using sentence frames: "When I substitute x = ___, I get ___" "This is extraneous because ___"	I can explain why extraneous solutions occur in rational equations: "Extraneous solutions happen when..." and systematically check all solutions using mathematical language.	I can predict when extraneous solutions might occur, explain the mathematical reasoning behind their existence, and analyze why certain algebraic operations can introduce invalid solutions.
LT8	I can recognize that identities are equations that are always true and identify simple examples using sentence frames: "An identity means ___" "This equation is true for all values because ___"	I can explain what mathematical identities are and distinguish them from conditional equations: "Identities are different from regular equations because..." using mathematical language.	I can analyze complex mathematical identities, explain their significance, and distinguish between identities, conditional equations, and contradictions in various mathematical contexts.

LT9	I can verify simple identities by substituting values or using basic algebraic steps with guided support using sentence frames: "I can show this is true by ___" "Both sides equal ___ when ___"	I can prove identities using algebraic manipulation and explain my reasoning: "I can rewrite the left side as..." "This proves the identity because..." using mathematical language.	I can construct rigorous proofs of complex identities, explain multiple proof strategies, and connect identity verification to broader algebraic reasoning principles.

Unit Description

Building on the "Polynomial Functions" unit, in this unit, students transition to working with rational functions, rational equations, and identities.

The unit begins with an introduction to rational functions as students consider situations they can model, such as when minimizing surface area or determining average costs. Students examine the asymptotic behavior of their graphs, which relates to the structure of the equations. Students continue to build on structure as they use polynomial division to rewrite rational expressions for the purpose of identifying the end behavior of the function. Students then focus on solving rational equations and making sense of how some methods can lead to possible solutions that are in fact not solutions (so-called "extraneous solutions").

In the final section, students study identities. Students sharpen their skills manipulating expressions while proving, or disproving, that two expressions are equivalent. The unit concludes with a return to geometric sequences first examined in the previous unit and, using a polynomial identity proved at the start of the section, students derive the formula for the sum of a finite geometric series before using the formula to solve problems.

Lesson Sequence	Learning Target	Success Criteria/ Assessment
Section A Rational Functions Lessons 1-4	**Learning Target 1** I can create and interpret rational functions that model real-world situations (such as surface area of cylinders with known volume) and explain what the function represents in context. **Learning Target 2** I can identify and interpret key features of rational functions from graphs and equations, including vertical asymptotes, horizontal asymptotes, and end behavior. **Learning Target 3** I can determine horizontal asymptotes and end behavior by analyzing the degrees of numerator and denominator, and by rewriting rational functions in the form f(x) = q(x) + r(x)/b(x). **Learning Target 4** I can interpret graphs of rational functions in context, explaining the meaning of asymptotes and other features in real-world situations, both orally and in writing.	**Lesson 1 Minimizing Surface Area** ● I can write a rational function to model different properties of cylinders. End of Unit Problem 7 **Lesson 2 Graphs of Rational Functions (Part 1)** ● I can identify a vertical asymptote from a graph or an equation of a rational function. End of Unit Problem 3 Check Point A Problem 1 **Lesson 3 Graphs of Rational Functions (Part 1)** ● I can identify a horizontal asymptote from a graph or an equation of a rational function. End of Unit Problem 3 Check Point A Problem 2 **Lesson 4 End Behavior of Rational Functions** ● I can find the end behavior of a rational function by rewriting it as f(x) = q(x) + r(x)/b(x) End of Unit Problem 3
Checkpoint A	*Responding to Student Thinking* **Problem 1:** Points to Emphasize - If students struggle to determine reasonable solutions from a situation, revisit the idea throughout the unit. For example, in the activity referred to here, when students find a solution, ask them about its meaning in the situation and whether it is reasonable.	
Section B Rational Equations	**Learning Target 5** I can solve equations involving rational expressions using strategic multiplication and other algebraic methods, and	**Lesson 5 Rational Equations (Part 1)** ● I can write rational expressions that represent averages to answer questions about the situation.

Lessons 5-7	determine whether solutions are valid or extraneous.. **Learning Target 6** I can create and solve rational equations to model real-world situations and use these models to answer questions about the context. **Learning Target 7** I can identify when extraneous solutions occur in rational equations and explain why they arise, both orally and in writing.	Check Point B Problem 1 **Lesson 6 Rational Equations (Part 2)** • I can write and solve equations with simple rational expressions on each side. Check Point B Question 1 **Lesson 7 Solving Rational Equations** • I know how to check for extraneous solutions to rational equations. End of Unit Problem 4 Check Point B Question 1
Checkpoint B	*Responding to Student Thinking* **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of solving rational equations. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work. **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of creating rational expressions and equations. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.	
ACC **Section C:** Polynomial Identities Lessons 8-11	**Learning Target 6** Calculate the solution to a system of polynomial equations. **Learning Target 7** I can understand, prove, and create polynomial identities to represent mathematical situations and justify general relationships in writing. **Learning Target 8** I can derive and apply the formula for the sum of the first n terms in a finite geometric sequence to solve problems. **Learning Target 9** I can interpret and explain results involving geometric sums in real-world contexts, both orally and in writing.	**ACC Lesson 8 Polynomial Identities (Part 1)** • I understand what an identity is in mathematics. End of Unit Problem 5 Check Point C Question 1 **ACC Lesson 9 Polynomial Identities (Part 2)** • I can justify why identities are true. End of Unit Problem 5 Check Point C Question 1 **ACC Lesson 10 Summing Up** • I understand why the geometric sum formula is true. End of Unit Problem 2 **ACC Lesson 11 Using the Sum** • I can use the geometric sum formula to solve problems. End of Unit Problem 2 Check Point C Question 2
Checkpoint C	*Responding to Student Thinking* **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of expanding polynomials to prove identities. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work. **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of finding the sum of a finite number of terms of a geometric sequence. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.	
End of Unit Assessment		

Unit Title
Unit 6 (iM A2 U4): Complex Numbers and Rational Exponents
Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards	Lesson	Standards
Lesson 1	HSA-CED.A.1	Lesson 8	HSA-REI.A.2 HSA-REI.D.11	Lesson 15	HSN-CN.A.2
Lesson 2	HSA-REI.B.4 HSA-REI.B.4.b	Lesson 9	HSA-REI.A.2 HSG-GMD.A.3	Lesson 16	HSA-REI.B.4.a HSA-REI.B.4.b
Lesson 3	HSF-IF.C.7.e HSN-RN.A.1 HSN-RN.A.2	Lesson 10	HSN-CN.A.1	Lesson 17	HSA-REI.B.4.b HSA-REI.D.11 HSN-CN.C.7
Lesson 4	HSF-IF.C.7.e HSN-RN.A.1 HSN-RN.A.2	Lesson 11	HSN-CN.A.1	Lesson 18	HSA-REI.B.4 HSN-CN.C.7
Lesson 5	HSN-RN.A.1 HSN-RN.A.2	Lesson 12	HSN-CN.A.1 HSN-CN.A.2	Lesson 19	HSA-REI.B.4.b HSN-CN.C.7
Lesson 6	HSA-REI.A.2 HSF-IF.C.7.b	Lesson 13	HSN-CN.A.1 HSN-CN.A.2	Lesson 20	HSA-CED.A.4 HSA-REI.B.4 HSF-BF.A
Lesson 7	HSA-REI.A.1 HSA-REI.A.2 HSA-REI.B.4.b	Lesson 14	HSN-CN.A.1 HSN-CN.A.2		

Essential Question(s)	Enduring Understanding(s)
• How do we extend mathematical systems to solve problems that previously had no solutions? • How do we perform arithmetic operations with complex numbers that don't exist on the traditional number line while maintaining consistent mathematical rules? • When do algebraic operations create extraneous solutions, and how do we determine when our mathematical solutions are valid in both algebraic and real-world contexts?	• When existing number systems prove insufficient, mathematicians create new number types while preserving familiar arithmetic properties and logical consistency • Fractional exponents provide algebraic notation for roots that maintains exponent rule consistency while connecting area, volume, and other geometric measurements to symbolic expressions • The introduction of *i* ensures that every quadratic equation has solutions, creating a mathematically complete system where polynomial equations always have the expected number of roots • Operations like squaring both sides of equations can generate extraneous solutions that satisfy the manipulated equation but not the original problem, requiring systematic verification of all solutions • Geometric interpretations (area/volume), algebraic notation (rational exponents), and visual models (complex plane) each provide unique insights into the same mathematical relationships • New numbers and operations must follow the same arithmetic properties (commutative, associative, distributive) as familiar numbers to maintain mathematical coherence and usefulness • Understanding where functions and operations are defined or undefined reveals both mathematical structure and real-world constraints in problem contexts

Demonstration of Learning	Pacing for Unit
CFA 1: Checkpoint A (Lessons 1-5) CFA 2: Checkpoint B (Lessons 6-9) CFA 3: Checkpoint C (Lessons 10-15) CFA 4: Checkpoint D (Lessons 16-19)	28 days (14 class periods) Lessons to Modify (from adaptation pack) • Optional Lessons: 1, 2, 9, 14, 16, 19, 20 • Any of the 7 optional lessons may be safely skipped if

End of Unit Assessment NOTES • Problem 1: for academic: Change to multiple choice keeping choices A, B and E and change C to 4^2 • Problem 5: change directions to "solve for all values of x" • Problem 6: Remove part a, change parts b and c to "solve for all values of x"	the Check Your Readiness assessment indicates that students are familiar enough with the concepts to engage with the other lessons.
Family Overview (link below)	**Integration of Technology:**
https://accessim.org/9-12-aga/algebra-2/unit-4?a=family	• Desmos Online Graphing Calculator
Unit-specific Vocabulary	**Aligned Unit Materials, Resources, and Technology (beyond core resources)**
Rational exponent, Square root, Cube root, nth root, Radicand, Radical expression, Radical equation, extraneous solution, Imaginary number, Imaginary unit (i), Complex number, Complex conjugate, Completing the square, Quadratic formula, Complex solutions, Real solutions, Non-real solutions, Discriminant, Multiple solutions, Simplest radical form, Rationalize, Perfect square, Perfect cube, Irrational number, Number system extension, Closure property	• Desmos Online Graphing Calculator • Pear Assessment (Edulastic) • IL Classroom Adapted Resources • Compasses • Internet-enabled devices • Rulers • Scientific calculators
Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions**
Computer Science & Technology • Complex numbers for 2D graphics rotations and transformations, digital signal processing for audio/image applications, and Fast Fourier Transform algorithms for data compression Music & Audio Production • Complex numbers for sound wave analysis and digital effects processing, rational exponents for musical frequency ratios and harmonic relationships, and synthesizer programming with complex oscillators Finance & Data Science • Compound interest calculations with rational exponents for different compounding periods, complex statistical models for risk analysis and market prediction, and normal distribution mathematics requiring square root operations	• Students think rational functions are just fractions with variables, incorrectly cancel non-factored terms, and forget domain restrictions that exclude values making denominators zero • Students believe graphs can never cross any asymptotes, don't understand the real-world meaning of asymptotic behavior, and confuse holes with vertical asymptotes • Extraneous Solutions Ignored • Students incorrectly apply distributive property to denominators See teacher's guide for specific misconceptions aligned to each lesson
Connections to Prior Units	**Connections to Future Units**
Relevant Unit(s) to review: • Grade 8 Unit 7: Exponents and Scientific Notation • Algebra 1 Unit 7: Introduction to Quadratic Functions • Algebra 1 Unit 8: Quadratic Equations Essential prior concepts to engage with this unit • Properties of Exponents • Square and Cube Roots • Solving quadratics by completing the square • Solving quadratics using the quadratic formula	The entirety of this course prepares students for success on the College Board SAT exam as well as success in Precalculus.
Differentiation through *Universal Design for Learning*	
Engagement • Connect rational exponents to familiar geometric contexts like area and volume calculations to make abstract concepts concrete • Build on familiar integer exponent rules to justify and extend understanding to fractional exponents • Address the conceptual challenge of imaginary numbers by introducing them as solutions to previously unsolvable equations • Scaffold learning progression from square and cube roots to rational exponents to complex number operations • Make learning goals transparent about extending the number system and solving all quadratic equations • Encourage mathematical discourse about why extraneous solutions arise when squaring both sides of equations	

Representation

- Present complex numbers through multiple sensory modalities using the imaginary axis, coordinate plane representations, and algebraic notation
- Ensure clear visual design for representing square roots, cube roots, and rational exponent notation
- Support vocabulary development for terminology (rational exponent, imaginary number, complex number, extraneous solution, principal root)
- Connect symbolic radical expressions to verbal descriptions and geometric interpretations of roots
- Activate prior knowledge by explicitly connecting rational exponents to familiar exponent rules and root operations
- Guide pattern recognition in complex number operations and highlight critical features like $i^2 = -1$

Action & Expression

- Provide multiple ways for students to demonstrate understanding through radical equations, complex arithmetic, or geometric applications
- Support planning strategies for choosing appropriate methods to solve quadratic equations including those with complex solutions
- Offer various tools including graphing technology, exponent rule organizers, and complex number operation templates
- Scaffold executive functions by providing organizational frameworks for connecting rational exponents to radical notation
- Create opportunities for students to express mathematical thinking about extending the number system beyond real numbers
- Guide students in monitoring progress as they verify solutions and identify extraneous solutions in radical equations

Supporting Multilingual/English Learners

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

MLR4: Information Gap

Used in Lesson 15 (Info Gap: What Was Multiplied?) where students must ask precise questions to determine what complex numbers were multiplied together

MLR5: Co-Craft Questions

Used in Lesson 12 (What Does This Path Mean?) where students generate mathematical questions about complex number representations on the complex plane

MLR7: Compare and Connect

Used in Lesson 18 (Solving All Kinds of Quadratics) where students compare different methods for solving quadratic equations with complex solutions

MLR8: Discussion Supports

Used extensively throughout multiple lessons (Lessons 1, 2, 3, 4, 6, 9, 11, 12, 13, 17, 18) to support rich mathematical discussions about exponents, radicals, complex numbers, and quadratic solving methods

Sentence Frames and Stems

Section A

- In this situation, the input of the function _____ represents _____. When the input is _____, the output is _____ and represents _____.
- The growth factor in this situation is _____ because ...
- Between _____ and _____, the _____ increased/decreased by _____.
- The function _____ represents this situation because ...

Section B

- The expression _____ in exponential/logarithmic form is equivalent to the expression _____ in logarithmic/exponential form because ...
- The exact solution to the equation _____ is _____.
- To solve the equation _____, first I _____, then I ...
- The logarithmic expression _____ represents this situation because ...

Section C

- I noticed that for the function _____, the value of “f” increases/decreases when “x’ increases/decreases.
- To solve the logarithmic equation _____, first I _____, then I ...
- The solution to the equation _____ is _____ because ...
- The constant e is/is not similar to pi because...

Section D

- Using the graph of the situation represented by the equation _____, I know ... because ...
- On the graph of the function _____, the point _____ represents ...
- The _____-intercept of the graph is _____ and represents ...
- The exact solution to the equation _____ is _____.
- The intersection of the two graphs represents ...

Section E

- Logarithms help reason about the acidity in different liquids because ...
- _____ has _____ moles per liter of hydrogen ion concentration, which is _____ times greater/less than _____ because ...
- The logarithmic scale is used to measure _____ (example: earthquake intensity) because ...

Supporting Multilingual/English Learners (CELP standards)

	Emerging	Expanding	Bridging
LT1	I can evaluate simple expressions with integer exponents using patterns and sentence frames: "2^3 means ___" "5^{-2} means ___" with visual supports showing repeated multiplication and division.	I can evaluate expressions with integer exponents and explain the rules: "Negative exponents mean..." "When I multiply powers with the same base..." using mathematical language.	I can evaluate complex expressions with integer exponents, apply exponent rules strategically, and explain why the rules work using mathematical reasoning.
LT2	I can calculate basic square and cube roots using perfect squares and cubes with visual supports: "$\sqrt{9}$ = 3 because 3 x 3 = 9" using sentence frames and number patterns.	I can calculate square and cube roots and explain my reasoning: "I know $\sqrt{25} = 5$ because..." and estimate non-perfect roots using mathematical language with support.	I can calculate various roots efficiently, estimate irrational roots, and explain different methods for finding radical values including approximation techniques.
LT3	I can write simple radicals as fractional exponents using patterns: "$\sqrt{x} = x^{\frac{1}{2}}$" "$\sqrt[3]{x} = x^{\frac{1}{3}}$" with guided examples and sentence frames: "The square root of x is the same as ___"	I can convert between radical and exponential notation and explain the equivalence: "$\sqrt[3]{x^2} = x^{\frac{2}{3}}$ because..." using mathematical language to connect the representations.	I can fluently convert between radical and exponential forms, explain why the equivalence holds, and apply this understanding to simplify complex expressions.
LT4	I can interpret simple fractional exponents with guidance: "$x^{\frac{1}{2}}$ means ___" "$x^{\frac{2}{3}}$ means ___" using visual models and sentence frames connecting to radicals.	I can interpret fractional exponents and explain their meaning: "The denominator tells us..." "The numerator tells us..." connecting to radical notation with mathematical language.	I can interpret complex fractional exponents, explain the general pattern $x^{\frac{m}{n}}$, and apply this understanding to evaluate and simplify expressions.__
LT5	I can interpret negative fractional exponents with step-by-step support: "$x^{\frac{-1}{2}}$ means ___" using sentence frames and connecting to positive fractional exponents and reciprocals.	I can interpret negative fractional exponents and explain the reasoning: "Negative means reciprocal, so $x^{\frac{-2}{3}}$ = ..." using mathematical language to connect multiple concepts.	I can work fluently with negative fractional exponents, explain why the rules work, and apply them to solve complex problems involving rational exponents.
LT6	I can identify that âˆšx means only the positive square root using sentence frames: "$\sqrt{16}$ = 4, not -4, because _" with visual supports showing the distinction	I can explain why the square root symbol represents only positive values: "The radical symbol means the principal root because..." and distinguish this from equation solutions.	I can explain the mathematical reasoning behind the principal root convention, distinguish between $\sqrt{x}$ and solutions to $x^2 = a$, and apply this understanding consistently.
LT7	I can solve simple equations by squaring or taking square roots with guided steps: "If $x^2 = 9$, then x = ___" using sentence frames and checking solutions with support.	I can solve equations involving squares and square roots and explain my method: "I square both sides because..." "I need to check for extraneous solutions because..."	I can solve complex equations involving squares and square roots, explain when different methods are appropriate, and analyze why extraneous solutions occur.
LT8	I can solve basic equations with cubes and cube roots using patterns: "If x^3 = 8, then x = ___" with sentence frames and visual supports showing the relationship.	I can solve equations with cubes and cube roots and explain the process: "Cube roots are different from square roots because..." using mathematical language.	I can solve complex equations involving cubes and cube roots, compare solution processes with square root equations, and explain why cube root equations behave differently.
LT9	I can solve simple radical equations by isolating the radical and squaring with step-by-step guidance using sentence	I can solve radical equations and explain my solution process: "I isolate the radical because..." "I must check my	I can solve complex radical equations using strategic methods, explain why checking is essential, and predict when

	frames: "First, I isolate ___" "Then I square both sides to get ___"	answer because..." using mathematical language with support__	extraneous solutions might occur.
LT10	I can identify that $\sqrt{-1}$ = i and work with simple multiples like 2*i*, 3*i* using sentence frames: "$\sqrt{-4}$ = 2*i* because ___" with visual supports on the imaginary number line.	I can represent $\sqrt{-1}$ and its multiples using *i* notation and explain why this is necessary: "We need *i* because..." connecting to the impossibility of real square roots of negatives.	I can work fluently with $\sqrt{-1}$ and complex radicals, explain the historical development of imaginary numbers, and connect to the complex number system.
LT11	I can use i to solve simple equations like $x^2 = -4$ with guided support using sentence frames: "Since $x^2 = -4$, then x = ___" with step-by-step examples.	I can solve equations that require imaginary solutions and explain when to use *i*: "This equation needs *i* because..." using mathematical language to justify the approach.	I can solve complex equations requiring imaginary numbers, explain why complex solutions are necessary, and connect to the fundamental theorem of algebra.
LT12	I can add complex numbers and calculate simple powers of i using patterns and sentence frames: "(3 + 2i) + (1 + 4i) = ___" "i^2 = ___" with visual aids.	I can perform operations with complex numbers and explain the processes: "To add complex numbers, I..." "Powers of i follow the pattern..." using mathematical language.	I can perform all operations with complex numbers fluently, explain the underlying mathematical principles, and recognize patterns in complex number arithmetic.
LT13	I can multiply simple complex numbers using the distributive property with guided steps: "(2 + i)(3 + i) = ___" using sentence frames and step-by-step support.	I can multiply complex numbers and explain my method: "I use FOIL and remember that i^2 = -1..." connecting to polynomial multiplication with mathematical language.	I can multiply complex expressions efficiently, explain different multiplication strategies, and connect complex multiplication to polynomial operations.
LT14	I can perform basic arithmetic operations on complex numbers with step-by-step guidance using sentence frames for each operation type with visual supports and pattern recognition.	I can perform all arithmetic operations on complex numbers and explain when each is used: "Division requires..." "Multiplication involves..." using mathematical language consistently.	I can perform complex arithmetic fluently, choose efficient methods strategically, and explain the mathematical principles underlying complex number operations.
LT15	I can identify real and imaginary parts when given simple information with guided examples using sentence frames: "If z_1z_2 = 12 + 5i, then the real part is ___"	I can determine real and imaginary parts from given information about complex numbers and their relationships using mathematical reasoning: "Since the product is..., then..."	I can solve complex problems involving unknown complex numbers, use algebraic reasoning to find parts, and explain sophisticated relationships between complex numbers.
LT16	I can solve quadratic equations using completing the square and the quadratic formula with step-by-step guidance using sentence frames for each method and visual supports.	I can solve quadratic equations using multiple methods and explain when to use each: "I use the quadratic formula when..." "Completing the square is helpful when..."	I can solve quadratic equations strategically, choose efficient methods, and explain the advantages and connections between different solution techniques.
LT17	I can find complex solutions by completing the square with guided steps using sentence frames: "When I get $x^2 = -4$, then x = ___" connecting to previous learning about i.	I can complete the square to find complex solutions and explain the process: "When the constant is negative after completing the square..." using mathematical language.	I can complete the square to find complex solutions in various forms, explain when this method is most useful, and connect to graphical interpretations.
LT18	I can use the quadratic formula to find complex solutions when the discriminant is negative with step-by-step support using sentence frames about each part of the formula.	I can apply the quadratic formula for complex solutions and explain when this occurs: "Complex solutions happen when the discriminant..." using mathematical language.	I can use the quadratic formula for complex solutions efficiently, predict solution types from the discriminant, and explain the mathematical significance of complex solutions.
LT19	I can identify when quadratic equations have complex solutions and find them using either method with guided support and sentence frames connecting discriminant to solution type.	I can find complex solutions to quadratics using appropriate methods and explain my choice: "I chose this method because..." connecting discriminant analysis to solution strategies.	I can find complex solutions efficiently using strategic methods, analyze the nature of solutions before solving, and explain the complete solution process including verification.

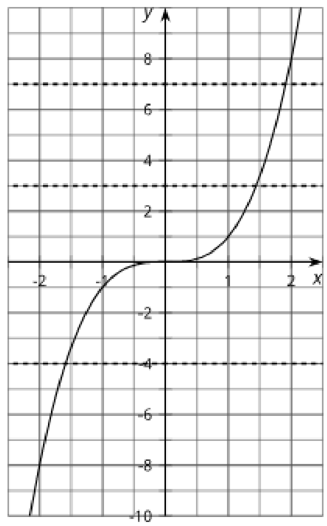
Unit Description

In this unit, students use what they know about exponents and radicals to extend exponent rules to include rational exponents, solve equations involving squares and square roots, develop an understanding of complex numbers, and solve quadratic equations that include complex roots.

The unit opens with an optional review of exponent rules before using those rules to justify why $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ when a and b are integers and x is positive. The new rule leads to an exploration of square and cube roots as solutions to equations of the form $x^2 = c$ or $x^3 = c$. In particular, students learn that positive numbers have two square roots and that $\sqrt{c}$ represents only the positive root.

Students explore equations involving square and cube roots to recognize that squaring each side of an equation can introduce solutions that are not solutions to the original equation.

Note that there are claims in the unit like, "When a square is equal to a negative number, there are no solutions," which would be more precisely stated as "When a square is equal to a negative number, there are no real number solutions." However, until students learn about imaginary numbers in the next section, it doesn't make sense to make this distinction.



The third section introduces imaginary and complex numbers by proposing i as a solution to the equation $x^2 = -1$. Students explore the implications of this new type of number by representing it on an imaginary axis off of the real number line. This exploration includes adding imaginary and real numbers together to get complex numbers, and then adding and multiplying complex numbers.

Next, students use their new understanding of complex numbers to solve more quadratic equations. Finally, there is an optional exploration of data representation on a map using circles with area proportional to the data, in which students must use their understanding of square roots to find the radii of the appropriate circles.

Lesson Sequence	**Learning Target**	**Success Criteria/ Assessment**	**Resources**
Section A Exponent Properties Lessons 1-5	**Learning Target 1** I can understand and justify why exponent properties hold for positive integers and use repeated reasoning to extend these properties to all integers and rational exponents. **Learning Target 2** I can comprehend the meaning of radicals (square roots, cube roots) and justify the equivalence between radical notation and rational exponents $(b^{\frac{1}{n}} = \sqrt[n]{b} \text{ and } b^{\frac{m}{n}} = \sqrt[n]{b^m})$ **Learning Target 3** I can determine exact side lengths using radicals, use graphs to estimate values of expressions with rational exponents, and represent numbers using both radical and exponential notation.	**Lesson 1 Properties of Exponents** • I can evaluate expressions with integer exponents. Check Point A Question 1 End of Unit Question 1 **Lesson 2 Square Roots and Cube Roots** • I can calculate square and cube roots. End of Unit Question 1 **Lesson 3 Exponents That Are Unit Fractions** • I can write square and cube roots as exponents. Check Point A Question 1 **Lesson 4 Positive Rational Exponents** • I can interpret exponents that are fractions. Check Point A Question 1 End of Unit Question 1 **Lesson 5 Negative Rational Exponents** • I can interpret exponents that are negative fractions.	
Checkpoint A	*Responding to Student Thinking* **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of writing expressions using rational exponents and radicals. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.		

<table>
<tr><td>Section B
Solving Equations with Square and Cube Roots

Lessons 6-9</td><td>Learning Target 4
I can explain the difference between square roots (which denote only positive values) and cube roots (which exist for all real numbers), and justify why equations involving these have different numbers of solutions.

Learning Target 5
I can solve equations involving square roots, cube roots, and other radicals, explaining my solution methods and using graphs to represent solutions.

Learning Target 6
I can explain why squaring or taking roots of equations may change the solution set, create radical equations with specified numbers of solutions, and identify extraneous solutions.</td><td>Lesson 6 Squares and Square Roots
• I understand that the square root symbol means the positive square root.
Lesson 7 Inequivalent Equations?
• I can solve equations by squaring or finding square roots.
Checkpoint B Question 1
End of Unit Question 3
Lesson 8 Cubes and Cube Roots
• I can solve equations by cubing or finding cube roots.
Checkpoint B Question 1
End of Unit Question 6
ACC Lesson 9 Solving Radical Equations
• I can solve equations with radicals in them.
Checkpoint B Questions 1 and 2
End of Unit Question 5</td></tr>
<tr><td>Checkpoint B</td><td colspan="2">Responding to Student Thinking
Problem 1: Press Pause: By this point in the unit, there should be some student mastery of finding the number of solutions for an equation. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.
Problem 2: Press Pause: By this point in the unit, there should be some student mastery of solving radical equations. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.</td></tr>
<tr><td>Section C
A New Kind of Number

Lessons 10-15</td><td>Learning Target 7
I can justify why $x^2 = -1$ has no real solutions using graphs, understand that i represents √(-1), and comprehend that complex numbers have the form a + bi where a and b are real numbers.

Learning Target 8
I can add, subtract, and multiply complex numbers, represent results in the form a + bi, and justify equivalence of complex expressions using the fact that $i^2 = -1$.

Learning Target 9
I can represent complex numbers and their operations on the complex plane, determine powers of i using patterns, and analyze what information is needed to find factors of complex number products.</td><td>Lesson 10 A New Kind of Number
• I can represent $\sqrt{-1}$ and multiples of it.
Lesson 11 Introducing the Number i
• I can use i to solve equations.
End of Unit Question 3 and 7
Lesson 12 Arithmetic with Complex Numbers
• I can add complex numbers and calculate powers of imaginary numbers.
Checkpoint C Question 1
End of Unit Question 4
Lesson 13 Multiplying Complex Numbers
• I can multiply complex numbers.
Checkpoint C Question 1
End of Unit Question 4
ACC Lesson 14 More Arithmetic with Complex Numbers
• I can do arithmetic with complex numbers.
Checkpoint C Question 1
End of Unit Question 4
Lesson 15 Working Backward
• I can find real and imaginary parts of complex numbers if I know enough about the numbers and their product.
Checkpoint C Question 1</td></tr>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
Problem 1: Press Pause: By this point in the unit, there should be some student mastery of complex number arithmetic. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.</td></tr>
<tr><td>Section D</td><td>Learning Target 10</td><td>Lesson 16 Solving Quadratics</td></tr>
</table>

Solving Quadratics with Complex Numbers Lessons 16-19	I can solve quadratic equations using completing the square and the quadratic formula, and explain my reasoning in writing for both real and complex solutions. **Learning Target 11** I can calculate complex solutions to quadratic equations, represent them in the form c ± di, and explain when and why solutions are non-real using both algebraic and graphical reasoning. **Lesson Target 12** I can create quadratic equations with specified solution types (real or non-real), critique solution methods used by others, and justify whether radical expressions are equivalent.	• I can solve quadratic equations by completing the square or by using the quadratic formula. Checkpoint D Question 1 End of Unit Question 3 and 7 **Lesson 17 Completing the Square and Complex Solutions** • I can find complex solutions to quadratic equations by completing the square. End of Unit Question 7 **Lesson 18 The Quadratic Formula and Complex Solutions** • I can find complex solutions to quadratic equations by using the quadratic formula. Checkpoint D Question 1 End of Unit Question 7 **Lesson 19 Real and Non-Real Solutions** • I can find complex solutions to quadratic equations CheckPoint D Question 1 End of Unit Questions 3 and 7
Checkpoint D	*Responding to Student Thinking* **Problem 1:** Press Pause: By this point in the unit, there should be some student mastery of solving quadratic equations. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work. **Problem 2:** Press Pause: By this point in the unit, there should be some student mastery of the number and type of solutions for quadratic equations. If most students struggle, make time to revisit related work in the practice problems referred to here. See the Course Guide for ideas to help students re-engage with earlier work.	
Section E Let's Put It to Work Lesson 20		**ACC Lesson 20 Map Data** • I can represent data on a map using appropriate sized circles.
End of Unit Assessment		

This is a living document.
For the most recent electronic copy of this document scan the QR code